FACULTY OF ECONOMICS AND BUSINESS

Construction of a quantitative macro-economic reverse stress test for a simulated bank

Aster Duflou

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Thesis submitted to obtain
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MASTER IN DE TOEGEPASTE ECONOMISCHE WETENSCHAPPEN:
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Major Risk and finance

Promoter: Prof. Dr. Kristien Smedts
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FACULTY OF ECONOMICS AND BUSINESS

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Reverse stress tests in the banking sector refer to the identification of the most likely scenarios which lead to a bank's default. Despite the regulatory requirement of conducting these tests, no quantitative standard method is imposed so far. This paper aims to contribute in filling this gap by constructing a quantitative reverse stress testing framework which focuses on credit risk and implementing it on a simulated Belgian savings bank. In this study, reverse stress test scenarios are defined as combinations of macro-economic factors which result in a CET1 ratio below the mandatory minimum. Several statistical and financial models are employed: principal component analysis serves as a dimension reduction technique necessary to preserve computational tractability, an extended CreditMetrics^{TM} model allows for the simulation of the credit migrations of obligors and a dynamic lag model is applied to predict future default probabilities of mortgage loans. Additionally, extreme value theory and copula functions are employed to compute the scenarios' probability. After describing the reverse stress test's main results, several important model risks and limitations of the framework are discussed.

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Table 0.1. Overview of abbreviations

Abbreviation	In full
AIC	Akaike Information Criterion
A-IRB	Advanced Internal Ratings Based
ADF	Augmented Dickey-Fuller
BCBS	Basel Committee on Banking Supervision
BIC	Bayesian Information Criterion
BIPRU	Prudential sourcebook for Banks, Building Societies and Investment Firms
CEBS	Committee of European Banking Supervisors
CET	Common Equity Tier
CICR	Central Individual Credit Register
CRD	Capital Requirements Directive
CRR	Capital Requirements Regulation
DSB	Dirk Scheringa Beheer
df	Degrees of freedom
EBA	European Banking Authority
ECB	European Central Bank
EFRAG	European Financial Reporting Advisory Group
EL	Empirical Likelihood
EP	European Parliament
ES	Expected Shortfall
ESRB	European Systemic Risk Board
EVT	Extreme Value Theory
F-IRB	Foundation Internal Ratings Based
FCA	Financial Conduct Authority

FSA	Financial Services Authority
GPD	Generalized Pareto Distribution
HPI	House Price Index
i.i.d.	independent and identically distributed
IRB	Internal Ratings Based
JB	Jarque-Bera
KMV	Kealhofer, McQuown, and Vasicek
KS	Kolmogorov-Smirnov
LCR	Liquidity Coverage Ratio
LGD	Loss Given Default
ML	Maximum Likelihood
MLE	Maximum Likelihood Estimator
MRL	Mean Residual Life
NSFR	Net Stable Funding Ratio
PC	Principal Component
PCA	Principal Component Analysis
PD	Probability of Default
PRA	Prudential Regulation Authority
RST	Reverse Stress Test
RSTS	Reverse Stress Test Scenario
RW	Risk Weights
RWA	Risk Weighted Assets
STA	Standard (Approach)
S&P	Standard and Poor’s
VaR	Value at Risk

Table 0.2. Description of symbols.

Symbol	Description
$C_T(u)$	Estimated copula function under H_0, evaluated at value u
$c_{q,j}$	q-th coefficient of j-th principal component
$C_{\widehat{\theta_T}}(u)$	Empirical copula, evaluated at value u
$C(.)$	Cumulative copula function
$CET1_{1\%}$	1% quantile of the CET1 ratio's distribution
$d_i(t)$	Number of defaults of i-rated corporations in period t
$d_{i,j}$	Euclidean distance between reverse stress test scenario i and j
D_{total}	Total Euclidean distance between selection of reverse stress test scenarios
EL	Expected Loss
$E(new\ mortgage\ loans\ value \vert \omega)$	Expected value of the mortgage loans portfolio conditional upon a given scenario
$E\left(PD_{mortgage} \middle\vert \omega\right)$	Expected value for $PD_{mortgage}$ conditional upon a given scenario
$F(y)$	Cumulative distribution function of unspecified type, evaluated at point y
gdp	Realization of Belgian Domestic Product growth in period t
$GDPgrowth(t)$	Belgian Gross Domestic Product growth in period t
gdp_H, gdp_L	Higher resp. lower bound of a scenario w.r.t. its dimension GDP growth

$G_{\xi,\beta}(y)$	Generalized Pareto distribution, i.e. cumulative distribution in right tail conditional upon breaching threshold u, with scaling parameter β and shape parameter ξ, evaluated at point $u + y$
$G(.)$	Inverse cumulative distribution function
hpi_H, hpi_L	Higher resp. lower bound of a scenario w.r.t. its dimension ΔHPI
H_0	Null-hypothesis
K, k	Maximum number of lags allowed to be included (resp. actually included) for ϵ_t in dynamic lag model for $PD_{mortgage}$
k	Number of parameters required by copula function
L, l	Maximum number of lags allowed to be included (resp. actually included) for $GDPgrowth_t$ in dynamic lag model for $PD_{mortgage}$
L_n	Original loan value of obligor n
$L_{n,default}$	Loan value of obligor n after default
l_i	Log-likelihood function of an initially i-rated obligor
M, m	Maximum number of lags allowed to be included (resp. actually included) for ΔHPI_t in dynamic lag model for $PD_{mortgage}$
m	Time to maturity
m_{ij}	Unadjusted migration probability from rating i to rating j
$m_{ij,adj}$	Migration probability from rating i to rating j, adjusted for rating withdrawals

N, n	Maximum number of lags allowed to be included (resp. actually included) for $\Delta UNEMPL_t$ in dynamic lag model for $PD_{mortgage}$
n	Dimensionality of scenarios
$N_i(t)$	Number of i-rated corporations in period t
$N(.)$	Normal cumulative distribution function
p	Number of principal components selected
P	Correlation matrix
$Pr(.)$	Probability
pc_j	Realisation of j-th principal component of the risk-free interest rate term structure
PC_j	j-th principal component of the risk-free interest rate term structure
$pc_{j,H}$, $pc_{j,L}$	Higher resp. lower bound of a scenario w.r.t. its dimension PC_j
$PD_{category}$ $category \in \{investment, speculative\}$	Default probability of a corporation in a given credit rating category
$PD_{mortgages}$	Default probability of the mortgage portfolio
PD_y	Corporate default probability in year y
PD_{y,Q_i}	Corporate default probability in year y, quarter i
q_i	Default probability of i-rated corporate obligor
R	Correlation in the RWA formula according to the IRB approach
$]R_{i,k}, R_{i,k+1}]$	Asset return threshold to be breached to migrate from i-rating to k-rating
RWA	Risk weighted assets

RWA_{cash}	Risk weighted cash
$RWA_{financial\ assets}$	Risk weighted financial assets
$RW_{corporate}$	Risk weight of the corporate obligor portfolio
$RW_{mortgage}$	Risk weight of the mortgage portfolio
$r_m(t)$	Risk-free interest rate for maturity m in period t
S	Number of observations in estimation sample in procedure to compute RMSPE
S_T	Cramér/von Mises test statistic
t	Time index
T	Total number of periods included in data sample
$TREND_t$	A time series representing the trend over time
$u = (u_1, \ldots, u_n) \in [0,1]^n$	Pseudo-observations
u_R, u_L	Threshold value in right resp. left tail of distribution
$unempl_H, unempl_L$	Higher resp. lower bound of a scenario w.r.t. its dimension $\Delta UNEMPL$
$VaR_{99\%}$	Value at risk at 99% confidence level
w_i	Withdrawal probability of an i-rated company
$X_{i,Max}, X_{i,min}$	Highest (lowest) value considered for the scenarios' dimension corresponding to the risk factor X_i
z	Realization of systematic risk factor in period t
Z(t)	Systematic risk factor in period t
z_H, z_L	Higher resp. lower bound of a scenario w.r.t. its dimension Z
α	Confidence level
$\Delta HPI(t)$	First differences of Belgian house price index in period t

$\Delta r_m(t)$	First differences of risk-free interest rate for maturity m in period t
$\%\Delta r_m(t)$	Relative differences of risk-free interest rate for maturity m in period t
$\Delta UNEMPL(t)$	First differences of Belgian unemployment rate in period t
ϵ_t	Residual of first part of the model to predict $PD_{mortgage}$, at period t
$\epsilon_n(t)$	Idiosyncratic risk of corporate obligor n in period t
δ_t	Residual of second part of the model to predict $PD_{mortgage}$, at period t
λ_j	Eigenvalue of j-th principal component
μ_i	Default probability of initially i-rated corporation
μ_{X_i}	Sample average of risk factor X_i
$\rho_{i,GDP}$	Asset return sensitivity of an initially i-rated corporation with regard to GDP growth
ρ_{i,PC_j}	Asset return sensitivity of an initially i-rated corporation with regard to PC_j
$\sqrt{\rho_{i,Z}}$	Asset return sensitivity of an initially i-rated corporation with regard to Z
$\sqrt{1-\rho_{i,Z}}$	Asset return sensitivity of an initially i-rated corporation with regard to ϵ
σ_{X_i}	Standard deviation of risk factor X_i
$\phi(.)$	Normal density distribution
$\Phi(.)$	Cumulative normal density distribution
ω	A certain scenario

Note that the symbols $GDP, HPI, UNEMPL$ are sometimes used to denote the GDP growth and first difference in HPI and unemployment rate with the aim to simplify notation.

1. Introduction

The financial crisis of 2007-2009 has proven that forecasts of banks' potential losses made by regular risk measures, such as Value at Risk (VaR) and Expected Shortfall (ES), are insufficiently reliable. More specifically, they failed to correctly take into account the possible occurrence of extreme tail risks. Hence, they did not succeed in pointing out that first, preventive actions should have been undertaken to avoid the large unexpected losses in the banking sector, and second, which specific actions would have been appropriate. Consequently, supervisory authorities emphasized the importance of the already required regular stress tests and adopted reverse stress tests (RSTs) as a regulatory requirement (BCBS, 2010; CEBS, 2010; FSA, 2009, 2011).

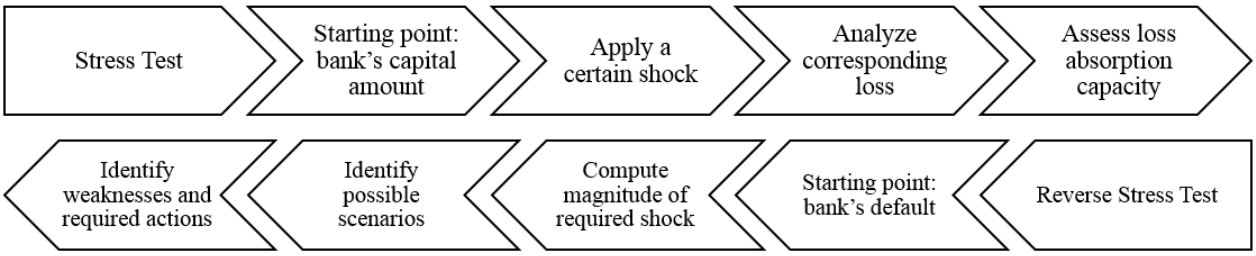

Figure 1.1. Stress Test and Reverse Stress Test.
(Adapted from: Farid, 2016)

The main differences between these two types of stress tests are illustrated in Figure 1.1. In the upper chain it is shown how regular stress tests take a given financial or economic shock[1], which is based on expert judgement or historical events, as a starting point and proceed by assessing the potential impact of this scenario on the bank's health, a.o. solvability, profitability, liquidity. The lower chain in Figure 1.1 demonstrates that RSTs, on the other hand, start from a loss of a given magnitude, typically a loss that leads to a bank's default[2], and then work backwards with the aim to identify the scenarios which could result in a loss equal to or exceeding this threshold. However, focus lies on scenarios that are not only sufficiently severe, but also plausible. Therefore, the next step is to rank these scenarios according to their relative[3] probability, allowing for the most likely scenarios to be uncovered.

[1] Examples are sudden, drastic changes in global equity prices, exchange rates, oil and commodity prices, sovereign credit spreads, residential and commercial property prices, etc. (ESRB, 2016).

[2] Note that the chosen definition of default is important as different default definitions may result in different RSTS identifications. The default definition adopted in this paper is specified in Section 2.3.

[3] In Section 2.5.2, an explanation is provided as to why the methodology applied in this paper only allows for the computation of a scenario's relative probability, i.e. as compared to other scenarios' likelihood. Note that this is sufficient as the FSA does not

The use of RSTs has various advantages. First, similar to regular stress tests, they highlight and help firms understand formerly unknown vulnerabilities, hence allowing for improved risk management and targeted mitigating actions (EBA, 2015; FSA, 2009). Second, this approach forces financial institutions to anticipate and look beyond the problems they have already incurred in the past or that seem likely to occur. Now, they are obligated to take into account the correlation between tail events, which are situations that only have a small chance of happening. RSTs can overcome disaster myopia and avoid the misleading sense of security in case forward stress tests result in manageable effects (CEBS, 2010). Third, the weaknesses identified by RSTs can be considered as starting point for resolution plans (FSA, 2009). Lastly, RSTs do not solely rely on expert judgement for scenario selection. Instead, these scenarios, which are in fact combinations of relevant risk factors that could lead to default, are identified automatically, hence reducing the arbitrariness in the scenario selection (Grigat and Caccioli, 2017).

Along with these advantages of deeper insight with regard to potential risks and solutions, the use of RSTs implies two key disadvantages. First, the conduct of RSTs is complex. It requires quantitative skills and an extensive understanding of financial markets. More specifically, appropriate RSTs need to be developed on an individual bank's basis as different banks may be affected by other risk factors, i.e. (macro-economic) factors which influence the bank's health and profitability. Understanding which risk factors need to be included in the analysis and how the risk factors relate to the bank's activities and portfolios calls for profound financial and statistical knowledge. A second disadvantage is the computational intensity required in the case of a realistic bank portfolio. This is due to the inclusion of many different types of assets and liabilities, the values of which may be influenced by a large number of risk factors. The total set of considered factors determines the number of dimensions of the scenarios to be identified, since these scenarios are in fact combinations of specific realizations of the considered risk factors. Any increase in dimensionality leads to a combinatorial explosion, i.e. an exponential increase in possible combinations. Therefore, a dimension reduction technique is imperative and the proposed framework must be developed in such way that its practicality lasts when considering more complex portfolios (Grundke and Pliszka, 2017).

require institutions to assess the scenarios' likelihood in absolute quantitative terms. It is considered sufficient to rank the identified scenarios instead of calculating absolute probabilities (FSA, 2011).

1.1. Regulation on RST

The benefits of RSTs have drawn the attention of financial service regulators. In response to the collective lack of imagination that became apparent due to the financial crisis, the FSA[4] pioneered in putting forward an RST regime. This new requirement became officially effective as from December 14, 2010 (FSA, 2009). In this regime, banks, insurers, building societies and a number of BIPRU investment firms are required to implement RSTs proportionally, meaning that whereas RSTs in small organizations can be exclusively qualitative, larger and more complex organizations are expected to conduct both qualitative and quantitative RSTs (FSA, 2009). The new RST requirement is included in the FSA's integrated stress testing framework[5] and does not represent a replacement of the already existing regular (forward) stress tests, but a complement to them. Moreover, Kilavuka (2013) argues that a self-perpetuating process can be developed by including the principal influencing parameters proposed in the outcome of the RSTs as an input in forward stress tests. In policy statement 09/20 the FSA stresses that the RST regime is proposed as a tool to improve risk management and business planning and not with the aim to determine capital requirements. Also, the results of RSTs need to be reported and validated by senior management or the board at least annually (FCA, 2017; FSA, 2009).[6]

Although the authorities oblige banks to conduct RSTs, no quantitative standard has been developed so far. To the question as to what quantitative analysis should be included in the submission, the FSA replied that quantitative analysis should be used to describe scenarios where appropriate and possible, but did not provide further details on which approach should be employed. Instead, the FSA states that no template for submission will be offered and that instead, firms are asked to develop their own format, tailored to their specific risk framework (FSA, 2011). The CEBS[7] (2010) provides similar statements about the lack of a standard for RST methodologies, the complementation of RSTs to regular stress tests, the concept of proportionality and the goal to represent a risk management tool rather than a method to compute capital requirements.

[4] The FSA (Financial Services Authority) is the predecessor of the FCA (Financial Conduct Authority) and the PRA (Prudential Regulation Authority). It had the responsibility over the regulation of the financial services institutions in the U.K. until 2013.

[5] Appendix 1 provides a more extensive description of the inclusion of RSTs in the FSA's integrated stress testing framework.

[6] More information on the degree of involvement required by the Board is provided in FSA FAQs (2011).
Additionally, Takano et al. (2013) note that bank executives' active involvement in stress testing is crucial for a successful implementation. It is the board's responsibility to impose collaboration of different business divisions with the aim of diminishing a stress event's effect. Consequently, stress testing would be less effective without this engagement.

[7] The CEBS (Committee of European Banking Supervisors) is the predecessor of the EBA (European Banking Authority). After its establishment by the European Commission in 2004 it acted as an independent advisory organization on banking supervision until 2010.

1.2. Related research

Not only is there no standard provided by the authorities, but also literature on the topic of RSTs is still scarce and, more specifically, only few proposals for quantitative RST methodologies have been formulated. Within the existing literature that covers quantitative RSTs, two domains can be distinguished. Whereas the first strand of literature discusses the identification of reverse stress test scenarios (RSTSs), the second strand focuses on the computation of the RSTSs' probability. Even though both strands take a different perspective, they have to deal with one common complication: the computational effort driven by the portfolio's degree of complexity. Consequently, both groups discuss the required application of dimension reduction techniques.

Examples in the domain of RSTS identification are Skoglund and Chen (2009), Grundke (2011), Grundke (2012), Grundke and Pliszka (2017) and Takano et al. (2013).

Skoglund and Chen (2009) pioneer the discussion of RSTS identification, proposing a measure based on the so-called Kullback information theory (Kullback, 1959) to develop a non-parametric method for extracting relative information of risk factors. The Kullback information measure gives an indication of which risk factors are important for determining profit and loss. This method is closely related, but superior, to the better known Euler risk contributions; whereas the latter can only be applied when a portfolio is simulated using a linear model, the Kullback information measure is also valid for non-linear simulation models. As the value of the instruments included in a portfolio is often a non-linear function of different risk factors, this Kullback information measure is valuable for RSTS identification. In the application of their theory on a sample portfolio consisting of equity derivatives, the relative information measure is computed for each of the underlying stocks. In other words, Skoglund and Chen (2009) define a scenario's dimensions as the risk drivers that determine the derivatives' value. However, no concrete explanation or example is provided as to how this theory can be implemented to identify RSTSs that are defined as a combination of macro-economic risk factor realizations.

This first segment of the literature dealing with quantitative RSTS identification is later on complemented by a series of suggestions by Grundke (2011, 2012) and Grundke and Pliszka (2017). The main idea of Grundke (2011) is the following. Instead of computing economic capital requirements for different risk types separately, and subsequently aggregating these requirements, integrated risk management techniques need to take into account mutual dependencies of different risks and their corresponding losses in order to accurately calculate the appropriate total capital requirements. Therefore, the authors claim that bottom-up approaches, that model how different risk types interact with one another on the individual risk factors and financial instruments level, serve as an adequate framework in order to carry out RSTs and especially, to compute the probabilities of the scenarios. However, two points of criticism are expressed by the authors themselves. First, bottom-up approaches are still relatively new and for some risk types, such as operational risk, no bottom-up approaches exist yet that take these risks into account. Second, no back-testing analyses of bottom-up approaches exist.

Extensions of this framework allow to account for more realistic banking characteristics. More specifically, Grundke (2012) proposes six modifications to the model introduced in his earlier publication (Grundke, 2011). First, time-varying bank rating is incorporated. More specifically, it is assumed that the return on the bank's assets influences the rating of the bank itself over the risk horizon. This characteristic is included in all further extensions, except for the last modification, which consists of a model reduction. Second, the contagion effects between the obligors are included in the model. The existance of these contagion effects implies that one obligor's default probability is not only influenced by systematic risk factors, but also by defaults of other obligors. Third, systematic recovery risk is added, which implies the relation between systematic risk factors and obligors' asset returns on the one hand and the obligors' recovery rates on the other hand. Fourth, the impact of lagged interest rate factors on obligors' asset returns is included. Fifth, time-varying sensitivities of obligors' asset returns toward systematic risk factors are accounted for. Lastly, a model reduction is proposed. This simplification of the original model assumes that the corporate loans' value does not change by varying rating changes, unless the obligor defaults.

Grundke and Pliszka (2017) start from this framework and, using U.S. data, they demonstrate the experimental implementation of RSTs. They consider a bank portfolio containing only fixed-income instruments and show how principal component analysis (PCA) can be used in order to reduce the number of relevant risk factors. The method proposed consists of 6 steps. First, a PCA is conducted of the risk-free interest rate's term structure in order to reduce the number of relevant macro-economic risk factors to consider. Second, a linear model of obligors' asset returns is estimated in function of these risk factors. These estimates of asset returns are used in step 4. Third, the marginal probability distributions of the risk factors are estimated and combined in a multivariate probability distribution. Fourth, asset return thresholds are calibrated, such that up- and downgrades of obligors' credit quality can be identified. Fifth, all scenarios are evaluated for a simplified portfolio, with assets and liabilities only consisting of zero-coupon bonds and deposits. Sixth, the most likely scenario which results in a loss exceeding the total amount of capital held, is identified.

Takano et al. (2013) present an enhanced firm-value model[8], in which the systematic risk factors are extended to incorporate macro-economic factors. This approach is closely related to Grundke and Pliszka (2017). The latter models the asset returns in function of various macro-economic factors and defines an obligor's transition from a certain credit rating grade to another by a breach of the modeled asset return through a pre-specified threshold. Similarly, Takano et al. (2013) model the firm-value in function of macro-economic factors and subsequently define an obligor's rating grade migration by a breach of its firm value through a threshold. Thereafter, they continue by combining the enhanced firm-value with the so-called Laplace inversion method, allowing to calculate the risk measures of a portfolio and the risk contributions of the different assets therein and to perform RSTs. The proposed methodology provides a unified framework for forward and reverse stress tests, which guarantees consistency between the results of both stress test approaches.

Having discussed the first strand of literature, it is imporant to note that RSTs do not stop after the identification of the scenarios which could render the financial institution's business model unviable. Instead, they also try to select the most likely RSTS. Therefore, the RSTSs' probability must be computed. This probability quantification is the focus of the second strand of literature. Considerable contributions in this respect were made by Glasserman, Kang and Kang (2012), Flood and Korenko (2013) and Kopeliovich, Novosyolov, Satchkov and Schachter (2015).

In Glasserman et al. (2012), the most likely scenarios, for which confidence intervals are constructed, are determined via empirical likelihood (EL). Each of these scenarios has to surpass a predefined level of loss in portfolio value. Through randomized resampling, alternative scenarios resulting in extreme losses are found. An important benefit of the EL estimator is that it does not rely on significant assumptions about the market factors' conditional distributions. Additionally, the shape of the confidence regions is flexible in capturing tail behaviour and skewness of market factors. However, this approach relies on a large sample size and a large loss level in order to obtain accurate results. The data requirements corresponding to this approach are thus a significant drawback, as in practice data on large losses are limited and therefore, supplementation by other methods is needed.

Whereas Glasserman et al. (2012) identify the most likely scenario conditional on bearing a loss greater than a given threshold, Flood and Korenko (2013) later on look for the scenario generating the largest loss for any given plausibility threshold. Flood and Korenko decide to search the surface

[8] In a firm value model the credit quality of an obligor is represented by a firm value.

of an ellipsoid that shows a predefined plausibility threshold, by making use of the eigenvectors of the covariance matrix. This method can be supplemented by a PCA in order to explain a large proportion of the variance. This is recommended, as the number of scenarios (or points on the ellipsoid) rapidly increases with the number of market risks taken into account. One point of criticism is that, since the method proposed by Flood and Korenko only identifies scenarios of a certain given level of plausibility, other combinations, which result in a similar or larger loss, are not considered.

Kopeliovich et al. (2015) develop a new method for RSTS selection by combining a PCA of the risk factors with Gram-Schmidt orthogonalization. The collection of identified scenarios should satisfy the following criteria: first, the scenarios' occurrence is not too unlikely. Second, they are maximally different from each other in some concrete sense. Third, they include the highest likelihood scenarios for the most important principal components (PC).

Despite these papers' significant theoretical contributions, Grundke and Pliszka (2017) point out two common drawbacks. First, these studies only consider market risk and ignore credit risk, and second, the approaches proposed become numerically intractable for portfolios of realistic complexity. Therefore, no realistic empirical implementation can be solely based on these approaches.

Whereas the previous two strands of literature propose either theoretical or empirically implementable methodologies for the conduct of quantitative RSTs by a single institution (i.e. as part of a micro-prudential analysis), one can also conduct a RST as part of a macro-prudential analysis[9]. This perspective is taken on by recent research by Grigat and Caccioli (2017). Starting from a given systemic loss, the paper subsequently proceeds by reverse engineering dynamics of financial contagion effects to identify the scenario of smallest exogenous shocks that could result in such a loss. To illustrate this, Grigat and Caccioli (2017) perform an RST analysis of a system consisting of 44 European banks, the interbank exposures of which are reconstructed by employing a so-called RAS algorithm. They show, first, how a ranking of banks according to their systemic importance can be determined by computing the distribution across banks of worst case shock sizes and, second, that this ranking can be employed to implement targeted capital requirement policies, which can enhance a system's robustness.

[9] Note, however, that this is not in correspondence with the FSA's integrated stress testing framework (FSA, 2009), which stipulates that RSTs are part of a firms' own stress testing, and not of the system-wide stress testing.

1.3. This paper's contribution

As mentioned previously, few proposed methodologies consider credit risk, and those that do, are not practically implementable when considering a more realistic bank portfolio. Furthermore, they solely focus on corporate obligors and omit the inclusion of residential mortgage loans in the bank's portfolio. This paper's contribution is to fill this gap in literature by proposing a quantitative RST methodology for a Belgian saving bank, the portfolio of which contains both corporate loans and residential mortgage loans. Due to the consideration of this different type of portfolio, scenarios will consist of realizations of additional macro-economic factors, such as the unemployment rate and the house price index, besides factors typically considered, e.g. GDP and interest rates.

The remainder of this paper is subdivided in five main parts. Section 2 provides a theoretical basis and describes the proposed quantitative framework for RST and the employed methodologies. Section 3 covers an experimental implementation of the framework from Section 2. First, I describe the different data used, the analysis of encountered data problems and the corresponding measures taken to overcome them. Afterwards, I discuss the results of the experimental implementation, i.e. the parameter estimations of the models and the RSTSs determined. Finally, Section 4 discusses the framework's limitations, which can be considered as topics for further research, and concludes.

2. Reverse stress testing framework

This section first provides an overview of the overall framework proposed to identify the most relevant RSTSs for banks for which the main activities consist of granting corporate and residential mortgage loans. The concrete balance sheet considered is described in Section 2.1. The methodology for simulating the value of corporate loans and assigning risk weights to these loans is largely based on the macro-economic approach by Grundke and Pliszka (2017). For the mortgage loans, default probabilities ($PD_{mortgage}$) are estimated using an adjusted version of the forecasting model suggested by Li (2014).

2.1. Simulated bank

As mentioned in Section 1, RSTs need to be developed on a bank-specific basis. Therefore, a choice has to be made about the type of bank on which the RST is carried out. As this paper focuses on credit risk rather than on market risk, a savings bank is considered instead of an investment bank. This, combined with the choice of considering Belgian data, leads to Argenta Spaarbank, a large Belgian savings bank in terms of balance sheet total (Febelfin, 2016).

Table 2.1. Stylized version of Argenta's balance sheet 2016 (in millions EUR).
(Adapted from: Argenta, 2016a)

Assets		**Liabilities and shareholders' equity**	
Cash	905.80	Deposits of credit institutions	1.40
Financial assets	7,241.10	Deposits of other than credit institutions	31,615.30
Loans and receivables	26,521.60	Debt securities including obligations	1,210.50
Corporate loans	3,460.00	Equity	1,841.30
Mortgage loans	23,061.60		
TOTAL ASSETS	34,668.50	TOTAL LIABILITIES	34,668.50

Table 2.1 shows a simplified summary of Argenta Spaarbank's balance sheet of 2016. The asset side consists of cash, financial assets and loans, the latter being subdivided in corporate loans (±13%) and mortgage backed loans (±87%). These loan portfolios are the most important asset types, as the focus of the RST presented in this paper is credit risk. The corporate loan portfolio is

assumed to consist of 346 groups of obligors, each of which represents a loan amount of €10 million. The obligors within each group are assumed to be subject to the same idiosyncratic risk ϵ_n (see Section 2.4.2). For simplicity it is assumed that the financial assets available for sale are constant and that all loans and deposits are contracted in euro, and thus any equity risk or currency risk is neglected. The liabilities side contains capital, deposits and obligations. Other components of a typical balance sheet (a.o. financial assets held for trading purposes, tangible fixed assets, provisions) are omitted for clarity and simplification. It is crucial to limit ourselves to considering a stylized portfolio, since a portfolio of higher complexity is influenced by a large number of risk factors, and hence requires a significant computational effort for the RSTS selection procedure. This is explained in more detail in Section 2.4.3, which discusses the dimension reduction technique.

2.2. Overview of framework

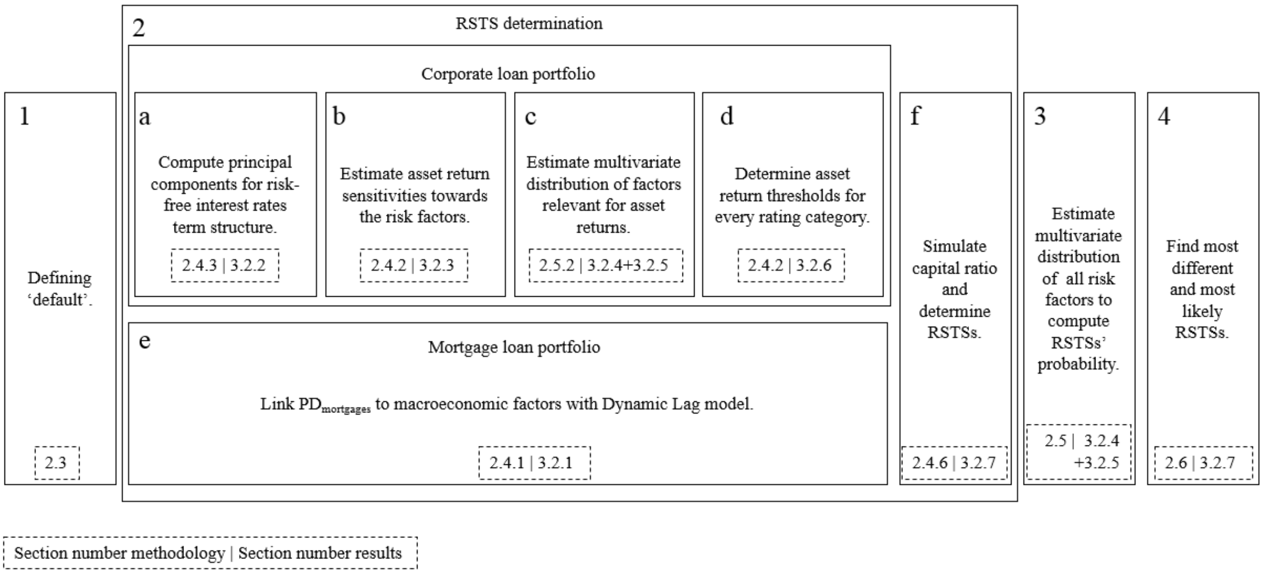

Figure 2.1. Overview of RST framework.

Figure 2.1 represents an outline of the proposed RST framework, which consists of four major components. The numbers provided at the bottom of each rectangle refer to the sections that respectively discuss the methodology used and the results obtained for this step. First, the point at which the bank defaults is defined in a quantitative manner. Second, the scenarios leading to the bank's default are determined. Steps 2.a – 2.d represent the different steps required by the CreditMetrics™ model, which simulates the credit migrations of corporate loans. In step 2.e the default probability of the mortgage loans portfolio is predicted using a Dynamic Lag (DL) model. After the parameters of the CreditMetrics™ model and the DL model are obtained, the two models are applied to simulate the capital ratio in step 2.f. In step 3 the probability of all scenarios for which this ratio falls below the default threshold is computed. Finally, in step 4, out of the ten most likely RSTSs three scenarios are selected such that they are maximally different from each other. The purpose of this last step is to maximize the information contained in the RSTSs submitted to the bank's top management; this reduces the risk of tunnel vision. Hence, it contributes to the achievement of RSTs' ultimate goal, which is to provide the necessary information to impose preventive actions to mitigate the extreme risks.

2.3. Default definition

RSTSs are defined by the EBA (2015) as those scenarios that result in a pre-defined outcome, e.g. scenarios which cause an institution's business model to become unviable, or which cause the bank to be likely to fail. However, no unique, quantitative definition of 'failure' or 'default' is provided in regulation. To answer the question of when one can speak of a business model failure, the FSA (2011) states that it may not be solely about inadequate financial resources. Additionally, the EBA (2015) refers to Article 32 in Directive 2014/59/EU of the European Parliament, in which again a qualitative approach is taken to define 'failure'. However, as this paper adopts a quantitative approach to RST, a concrete quantitative definition for 'default' is required. Therefore, it is defined as a breach of at least one of the capital requirements, which are presented in Table 2.2. These requirements are imposed by the Capital Requirement Directives (CRD) IV package, which is a supervisory framework reflecting the rules on capital measurements and standards outlined in Basel II and III. The interested reader is referred to Appendix 2 for a more extensive description of the Basel accords.

Table 2.2. Capital ratios and regulatory requirements.
(Adapted from: BCBS, 2013)

Capital ratio	Computation	Min. requirement	Conservation buffer	Min.requirement incl. buffer
CET1 capital ratio	$\frac{\text{Common Equity Tier 1}}{\text{Risk weighted assets}}$	4.5%	2.5%	7.0%
Tier 1 capital ratio	$\frac{\text{Tier 1 capital}}{\text{Risk weighted assets}}$	6.0%		8.5%
Total capital ratio	$\frac{\text{Total capital (Tier 1 + Tier 2)}}{\text{Risk weighted assets}}$	8.0%		10.5%

The third column of Table 2.2 specifies the minimum requirements for each ratio in 2016. However, additional capital conservation buffers and capital redefinitions are imposed over time[10] (column 4) and it is assumed that the bank considered in this paper wants to satisfy the fully phased-in minimum capital requirements (column 5) already ahead of the deadline in 2019. As this study considers a stylized bank balance, no distinction is made in types of capital. In other words, all capital on the balance sheet is assumed to be CET1 capital and all capital requirements in Table 2.2 consequently map to the same ratio. Default is then defined by a breach of the most stringent minimum requirement, i.e. $\frac{total\ capital}{RWA} = 10.5\%$.

The risk weighted assets (RWA) is the sum of asset exposures, where each asset type is multiplied with its corresponding risk weight. As the asset side of our stylized balance sheets consists of cash, financial assets, a corporate loan portfolio and a mortgage portfolio, the RWA can be computed by means of Equation (1).

$$RWA = RWA_{cash} + RWA_{financial\ assets} + RWA_{corp.loans} + RWA_{mort.loans} \quad (1)$$

Since $total\ assets = total\ liabilities$ and the bank's equity is assumed to be all CET1 equity, it holds that $CET1\ equity = total\ assets - liabilities$. Consequently, the CET1 ratio is computed as follows:

$$CET1\ ratio = \frac{total\ assets - liabilities}{RWA} \quad (2)$$

The computation of the RWA requires the choice of one out of three different methods: the Standard Approach (STA), the Foundation Internal Rating Based Approach (F-IRB) or the Advanced Internal Rating Based Approach (A-IRB). Further explanation about these methods is provided in Appendix 2.

Table 2.3. Risk weights for exposures to corporates, when STA is applied.[11]

	Exposure to corporates[12]						Exposure to cash[13]	Exposure to equity[14]
Credit quality step	1	2	3	4	5	6		
Risk weight	20%	50%	100%	100%	150%	150%	0%	100%

[10] For an overview and timeline of the phasing-in arrangements of conservation buffers and the phasing out of capital instruments that no longer qualify as non-core Tier 1 capital or Tier 2 capital, the reader is referred to BCBS (2013).

[11] Retrieved from: Part 3, Title 2, Chapter 2, Section 2, Article 122, EP, 2013.

[12] See Part 3, Title 2, Chapter 2, Section 2, Article 122, EP, 2013.

[13] See Part 3, Title 2, Chapter 2, Section 2, Article 134.2, EP, 2013.

[14] See Part 3, Title 2, Chapter 2, Section 2, Article 133.2, EP, 2013.

For the corporate loans component, the cash and the financial assets, the STA will be applied. Table 2.3 shows that, whereas this approach imposes fixed risk weights on cash and equity, corporate obligors are appointed different risk weights according to their credit quality. As this study relies on S&P credit ratings, the S&P credit rating scale is mapped to the credit quality steps of the STA approach. Table 2.4 describes how this mapping is executed.

Table 2.4. Mapping of S&P credit rating scale to credit quality step.
(Retrieved from: CEBS, 2006b)

Credit quality step	1	1	2	3	4	5	6
S&P credit assessment	AAA	AA	A	BBB	BB	B	CCC-C

As the balance sheet component of mortgage loans is considerably larger, the use of the F-IRB approach can be rationalized[15]. According to this method, the risk weights of mortgage loans are computed by Equation (3)[16],

$$RW_{mortgage} = \left(LGD \times N\left(\frac{1}{\sqrt{1-R}} \times G(PD) + \sqrt{\frac{R}{1-R}} \times G(0.999) \right) - LGD \times PD \right) \times 12.5 \times 1.06 \qquad (3)$$

where $N(x)$ denotes the normal cumulative distribution function of x; $G(z)$ represents the inverse cumulative distribution function of z; PD denotes the default probability; LGD stands for the loss given default and R is the coefficient of correlation, which equals 0.15 in case of residential mortgage loans[17].

2.4. Reverse stress test scenario determination

2.4.1. Predicting PD for residential mortgage portfolio

This section discusses the prediction of the default probability of the mortgage loan portfolio, which is required to compute the risk weight according to the F-IRB approach and the expected portfolio value for a given scenario. The expected loss amount of mortgage loans is computed as follows:

$$exposure\ value \times EL = exposure\ value \times PD \times LGD \qquad (4)$$

[15] Internal rating models must be developed and subsequently approved by a regulatory supervisor (CEBS, 2006a). Since this procedure costs a considerable amount of time and money, the use of the IRB approach will only pay off for portfolios of significant size and low probability of default.

[16] See Part 3, Title 2, Chapter 3, Section 2, Article 154, EP, 2013.

[17] See Part 3, Title 2, Chapter 3, Section 2, Article 154.2, EP, 2013.

where EL denotes the expected loss percentage. The expected value of the mortgage loan portfolio conditional on a given scenario ω can be obtained by Equation (5) (BCBS, 2005; EP, 2013).

$$\begin{aligned} E(new\ mortgage\ loans\ value|\omega) \quad = \quad & current\ mortgage\ loans\ value \\ & \times \left(1 - LGD \times E\left(PD_{mortgage}\middle|\omega\right)\right) \end{aligned} \tag{5}$$

Whereas the exposure value of the mortgage loans portfolio is assumed to be equal to the corresponding component reported on the balance sheet, as suggested by Tong, Mues, Brown and Lyn (2016), the expected default probability given a certain scenario and the loss given default (LGD) need to be set or estimated. The LGD is set to 10% as this is consistent with the regulatory lower bound of LGD[18] and with Argenta's Pillar 3 report of 2016.

Li (2014) provides an overview of different models available to generate a quantitative assessment of the probability of default for individual mortgage loans and mortgage portfolios, including a discussion of the advantages and disadvantages corresponding to each model. As the purpose is to assess the default rate of a loan portfolio as a whole, the most appropriate model according to Li is a linear regression analysis on log odds. This model estimates the log odds as a linear combination of the influencing factors selected and ensures that the dependent variable's value does not outpace the range [0,1], which is required as it represents a probability. The general model is described by Equation (6).

$$\ln \frac{PD_{mortgage}}{1 - PD_{mortgage}} \quad = \quad \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \ \beta_k X_k + \epsilon \tag{6}$$

where $PD_{mortgage}$ denotes the average default probability of mortgages in the portfolio and $(X_1, \dots, X_k)$ denotes a vector of influencing factors considered. After optimization of the coefficients in Equation (6), the model can be used to predict the log odds for certain values of the independent variables; subsequently, the implied predicted probability of default can be deduced.

In addition to this model, four categories of default determinants for residential mortgages are proposed by Li (2014), together with the empirical evidence found for every factor. Three of these categories, more specifically loan-, borrower- and property- specific variables, are considered as less relevant for this paper as their impact is averaged out in a well-diversified portfolio level study. The fourth category, macro-economic factors, such as unemployment rate, house price volatility and debt-to-GDP rate, remain important in a portfolio as they are affecting each and every mortgage loan in

[18] See Part 3, Title 2, Chapter 2, Section 3, Article 164.4, EP, 2013.

that portfolio. Including all macro-economic variables suggested by Li (2014) would lead to a considerable increase in the dimensionality of the scenarios and hence in the computational complexity of the framework. Instead, the variables with the most unambiguous empirical evidence in other literature are selected: GDP growth, unemployment and the house price index (HPI). For an extensive overview of previous research on the macro-economic factors' impact on default probability of mortgage loans and their results, the reader is referred to Appendix 2 of Li (2014). The empirical results of these research studies are consistent and intuitive: GDP growth and HPI have a negative impact on $PD_{mortgage}$, whereas the unemployment rate has a positive impact.

Attention should be paid when applying Li's suggested model as the data considered in this paper are time series. More specifically, the output of a linear model can only be interpreted correctly if both the dependent variable and all regressors are stationary. Appendix 3 discusses the concept of non-stationarity and spurious regression in more detail using the time series $\ln \frac{PD_{mortgage}}{1-PD_{mortgage}}$ and HPI as an example. In Section 3.2.1 it will be shown that $\ln \frac{PD_{mortgage}}{1-PD_{mortgage}}$ is trend-stationary. Therefore, to avoid spurious regression, it must be regressed on a trend term in a first model denoted by Equation (7). The residuals of this model will then be regressed on (stationary transformations of) the macro-economic factors suggested by Li (2014) and lags thereof (see Equation (8)).

$$\ln \frac{PD_{mortgage,t}}{1-PD_{mortgage,t}} = \beta_0 + \beta_1 TREND_t + \epsilon_t \tag{7}$$

$$\begin{aligned} \epsilon_t = {} & \alpha_0 + \alpha_{1,1}\epsilon_{t-1} + \cdots + \alpha_{1,k}\epsilon_{t-k} \\ & +\alpha_{2,0}GDPgrowth_t + \alpha_{2,1}GDPgrowth_{t-1} + \cdots + \alpha_{2,l}GDPgrowth_{t-l} \\ & +\alpha_{3,0}\Delta HPI_t + \alpha_{3,1}\Delta HPI_{t-1} + \cdots + \alpha_{3,m}\Delta HPI_{t-m} \\ & +\alpha_{4,0}\Delta UNEMPL_t + \alpha_{4,1}\Delta UNEMPL_{t-1} + \cdots + \alpha_{4,n}\Delta UNEMPL_{t-n} + \delta_t \\ & \text{with } k \in \{1, \dots, 4\}, l \in \{0, \dots, 4\}, m \in \{0, \dots, 4\}, n \in \{0, \dots, 4\}. \end{aligned} \tag{8}$$

Instead of arbitrarily choosing maximum lags (k, l, m and n in Equation (8)) for the different regressors, a loop is programmed that regresses all different combinations of maximum lags. The following constraint is imposed: if for a certain regressor the maximum lag considered is k, then all lags in between 0 and k are also included, since there is no intuitive or economical argument to exclude these lags from the regression. To ensure the model's compactness, the maximum lags that can be included are set to 4. It is however possible that different maximum lags (k, l, m, n) are included for ϵ, $GDPgrowth$, ΔHPI and $\Delta UNEMPL$. In total, 1080 $(= (4+1)(4+2)(4+2)(4+2))$ models are regressed and the corresponding Bayesian Information Criterion (BIC), Akaike

Information Criterion (AIC), R^2 and adjusted R^2 are saved. The 20 best models, i.e. the five best models according to each of the four criteria, are interactively compared.

2.4.2. Predicting corporate obligors' credit migrations and defaults

The STA of Basel III imposes different risk weights for differently rated corporate obligors. Therefore, the credit migrations of the bank's obligors must be modeled. Various methods are proposed so far: one can rely on rating agencies' judgement for corporate ratings, or historical methods (e.g. Altman Z-score) or market prices methods (Merton or KMV) can be applied to estimate the default probability, which are subsequently mapped into rating categories. However, the aim is to identify scenarios, defined as combinations of macro-economic risk factor realizations, that result in the CET1 ratio falling below 10.5%. Hence, the method is required to model the obligors' credit quality as a function of these risk factors.

Grundke and Pliszka (2017) propose to model rating migrations and defaults of individual obligors by applying an extended version of the CreditMetrics™ model, introduced by J.P. Morgan in 1997. There is one main difference between this CreditMetrics™ model and other well-known credit risk models, such as Vasicek's model or the Credit Risk Plus model; whereas these other models estimate the PD distribution and specific thresholds within these distributions, the CreditMetrics™ model instead estimates the asset return of an obligor n with credit rating i at time t using Equation (9) (Hull, 2012).

$$R_{n,i}(t) = \sqrt{\rho_{i,Z}} \times Z(t) + \rho_{i,GDPgrowth} \times GDPgrowth(t) + \sum_{j=1}^{p} \rho_{i,PC_j} \times PC_j(t) + \sqrt{1-\rho_{i,Z}} \times \epsilon_n(t) \quad (9)$$

where $Z(t)$ denotes an i.i.d. standard normal random variable, representing unobservable systematic credit risk, independent from all other macro-economic risk factors; $GDPgrowth(t)$ denotes the growth in GDP; $PC_j(t)$ is j-th principal component of the term structure of risk-free interest rates[19] and $\epsilon_n(t)$ denotes an i.i.d. standard normal random variable, representing idiosyncratic risk of obligor n; $\sqrt{\rho_{i,Z}}$, $\rho_{i,GDPgrowth}$, ρ_{i,PC_j}, $\sqrt{1-\rho_{i,Z}}$ denote the risk factor sensitivities with regard to respectively the unobservable systematic risk factor, GDP growth, the j-th principal component and the idiosyncratic risk.

[19] The computation and the reason for the use of principal components is explained in more detail in Section 2.4.3.

A number of studies, among which Rösch and Scheule (2007), Dunbar (2012) and Jakubik (2014), have already preceded this paper in the inclusion of the unobservable systematic credit risk, $Z(t)$, which stems from the conviction of numerous researchers (see Schwaab, Koopman and Lucas (2014, 2016) and references therein) that observable macro-economic factors and the idiosyncratic risk factor do not satisfy in explaining the obligors' asset return. The resulting model, given by Equation (9), can be regarded as an extended version of the one-factor model, which is often used when modeling credit quality (ECB, 2007; Hamerle, Dartsch, Jobst, Plank, 2011; Mager and Schmieder, 2009). The assumption that $Z(t)$ and $\epsilon(t)$ are standard normally distributed is widely used in these previous studies. Note, furthermore, that this model is similar to the well-known Merton model (1974). The main difference is that the CreditMetrics™ model allows for rating changes in addition to defaults (Gupton, Finger and Bhatia, 1997).

Other macro-economic factors, such as consumer price indices, could be included in Equation (9). However, as Section 2.4.3 explains, this would considerably reduce the computational tractability of the model and therefore this study opts to follow the assumption of Grundke and Pliszka (2017) that the factors above are most important in explaining the obligors' asset returns.

Under the assumption that the risk factor sensitivities in Equation (9) may differ per obligors' initial rating i (with $i \in \{AAA, \dots, C - CCC\}$), they can be estimated by maximizing the rating-specific log-likelihood, denoted by Equation (10).

$$l_i = \sum_{t=1}^{T} \ln \frac{\sqrt{1-\rho_{i,Z}}}{\sqrt{\rho_{i,Z}}} \frac{\phi(z_{thresh})}{\phi(\Phi^{-1}(\mu_i(t))} \qquad (10)$$

$$\text{with} \quad z_{thresh} = \frac{R_{i,8} - \rho_{i,GDP} GDP(t) - \sum_{j=1}^{p} \rho_{i,PC_j} pc_j(t) - \Phi^{-1}(\mu_i)\sqrt{1-\rho_{i,Z}}}{\sqrt{\rho_{i,Z}}}$$

where μ_i denotes the default probability of an obligor with initial credit rating i.

Due to unavailable data, this equation differs from the binomial log-likelihood used in Grundke and Pliszka (2017)[20] and is instead based on Demey, Jouanin, Roget and Roncalli (2004). A number

[20] Note that Grundke and Pliszka (2017) consider a log-likelihood function l_i with binomial shape:

$$l_i = \sum_{t=1}^{T} \ln \int_{-\infty}^{\infty} \binom{N_i(t)}{d_i(t)} \times q_i\left(z, \Delta\, log(gdp), c_1(t), \dots, c_p(t)\right)^{d_i(t)} \times \left(1 - q_i\left(z, \Delta\, log(gdp), c_1(t), \dots, c_p(t)\right)\right)^{N_i(t) - d_i(t)} \times \phi(z)dz$$

where $N_i(t)$ and $d_i(t)$ are the number of corporations and the number of defaults in rating category i at time t. Time series of these variables are however not publically available for Belgium or Europe. The asymptotic MLE serves as an approximating alternative in case the number of obligors in a rating category is large, while requiring only time series of PD data for each rating category, instead of $N_i(t)$ and $d_i(t)$.

of adjustments had to be made to the estimator suggested by the latter study. Appendix 4 provides a description of the differences between Equation (10) and the estimator proposed by Demey et al. (2004), together with the mathematical proof of equivalence of Equation (10) and the binomial log-likelihood used by Grundke and Pliszka (2017). As this proof relies on the assumption that N_i is large for all i, credit ratings AAA-BBB (respectively BB – C) are pooled into credit category 'investment grade' (respectively 'speculative grade'). Historical default and migration probabilities for these pools are computed as the weighted averages of the individual credit ratings' default and migration probabilities (see Appendix 9). Note that this is similar to the approach of Grundke and Pliszka (2017) in the sense that they also abandon the idea of investigating every single credit rating separately and instead use default data of larger groups that represent investment respectively speculative grade corporations.

Once the risk factor sensitivities are estimated, a credit migration of an initially i-rated obligor n to rating k (with $k \in \{AAA, \dots, CCC - C\}$) or a default (denoted by D) can be modeled by his asset return $R_{n,i}(t)$ falling within the range $]R_{i,k}, R_{i,k+1}]$. Consider the migration matrix in Table 2.5, the numbers of which denote the probability that a company with rating i, represented by a row name, migrates to a rating j, represented by a column name. The diagonal displays the probabilities of a company remaining in the same rating category. All numbers below (resp. above) the diagonal describe the likelihood of an upgrade (respectively downgrade) in rating.

Table 2.5. Average one-year migration matrix European companies 1981-2016.
(Retrieved from: S&P, 2016)

	AAA	AA	A	BBB	BB	B	CCC-C	Default
AAA	0.8714	0.1177	0.0065	0.0022	0	0	0.0022	0
AA	0.0030	0.8827	0.1083	0.0060	0	0	0	0
A	0.0001	0.0202	0.9131	0.0640	0.0020	0.0001	0	0.0004
BBB	0	0.0011	0.0461	0.9050	0.0421	0.0040	0.0011	0.0009
BB	0	0	0.0010	0.0603	0.8409	0.0883	0.0047	0.0047
B	0	0	0.0005	0.0043	0.0781	0.8361	0.0513	0.0297
CCC-C	0	0	0	0	0	0.1676	0.5079	0.3246

The data are adjusted for rating withdrawals as follows: $m_{ij,adj} = \frac{m_{ij}}{1-w_i}$ where m_{ij} is the unadjusted migration probability from rating i to j and w_i is the withdrawal probability of an i-rated company (Moody's, 2005).

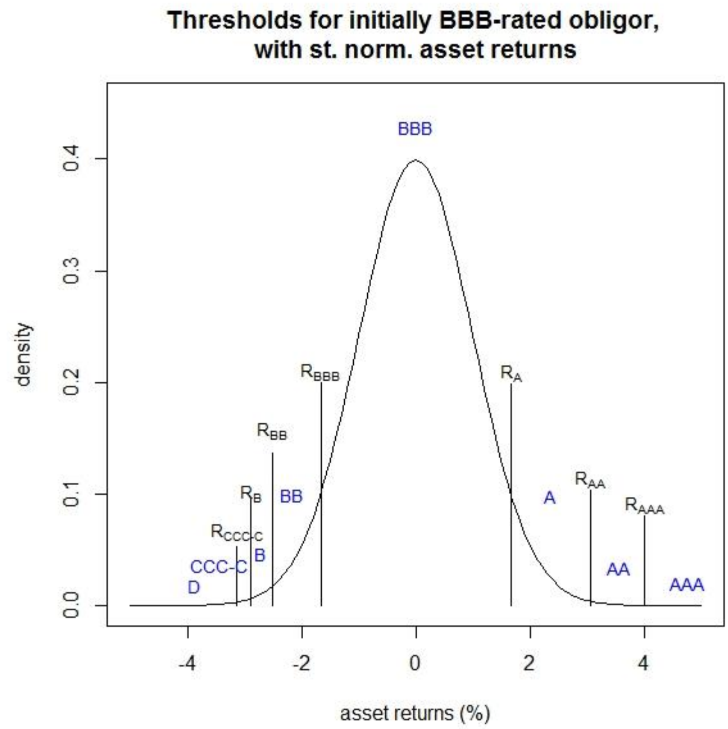

Figure 2.2. Asset return thresholds for BBB-rated obligor under assumption of standard normal asset returns.

Figure 2.2 presents the same information as the fourth row in Table 2.5 if one assumes that first, a rating migration can be represented by a breach of the obligor's asset return through a certain threshold, and second, that the asset returns are standard normally distributed. In spite of this second assumption being a simplification (see later in this section), this example can nevertheless be useful purely to explain how the thresholds, denoted by the vertical lines in Figure 2.2, are computed.

Consider the asset return threshold which needs to be breached by a BBB rated company in order to default. The fourth row in Table 2.5 describes that the probability of such an event equals 0.09%. Hence, the probability of breaching the threshold corresponding to a default, must be 0.09% as well. As the asset return distribution is assumed to be normal, one can now compute the threshold as the inverse of the cumulative standard normal distribution at percentile 0.09%. Equivalently, Table 2.5 shows that the probability of breaching the asset return threshold corresponding to a migration to a CCC-C rating, without breaching the default threshold, must be 0.11%. Thus, the according threshold must be the quantile corresponding to percentile 0.20% (=0.09% + 0.11%). Applying this reasoning for every rate migration of an initially BBB rated company results in the thresholds denoted in Figure 2.2.

However, this assumption of standard normally distributed asset returns is not necessarily correct, as the macro-economic risk factors in Equation (9) are not necessarily normally distributed. One must instead consider the empirical inverse distribution, which can be obtained by a Monte Carlo simulation in which a large number of samples from the multivariate probability distribution of $GDPgrowth, PC_1, \ldots, PC_p$ are drawn and the asset return is subsequently computed using Equation (9). The method to obtain this multivariate probability distribution of $GDPgrowth, PC_1, \ldots, PC_p$ is described in Section 2.5.

2.4.3. PCA as a dimension reduction technique

As briefly stated in the previous section, PCs of the term structure of risk-free interest rates are included in the CreditMetrics^{TM} model instead of the risk-free interest rates themselves. This section explains what principal component analysis (PCA) is and its importance in this paper's analysis.

Principal component analysis is a statistical technique that applies an orthogonal transformation to turn a dataset of observations consisting of correlated variables into a set of linearly uncorrelated variables, the so-called 'principal components' (PCs). The transformation ensures that the first PC accounts for as much variability of the given data set as possible. Next, other PCs are created, all capturing as much of the remaining variance of the original variables as possible, while additionally satisfying the constraint that they are orthogonal to the previously created PCs. Importantly, the number of resulting PCs is lower than the number of original variables in the dataset. This feature allows to apply PCA as a dimension reduction technique.

The computational intensiveness of the scenario selection procedure in RSTs increases with the number of risk factors considered. More specifically, for n risk factors, the scenarios that need to be identified in the inversion problem and for which the corresponding probability needs to be computed are also of dimensionality n. Realistic portfolios, consisting of many different types of assets and instruments, are influenced by many different risk factors. Hence, it is important that the RST framework is established in such a way that it remains numerically tractable for an increasing number of risk factors (Grundke and Pliszka, 2017).

In this paper, PCA is employed in order to reduce the number of risk factors while capturing as much variance from the interest rates' term structure as possible. A number of rules of thumb have been proposed for selecting the number of PCs to retain. First, the Kaiser-Guttman rule (suggested by Guttman (1954), and adjusted by Kaiser (1960, 1961)) suggests to retain all PCs with eigenvalues larger than 1[21]. Second, the scree plot, which is a plot of the PC's eigenvalues from largest to smallest, can be examined and the appropriate number of PCs is determined as the point where the plot reaches an "elbow" and after which the eigenvalues become very small. Third, PCs are added until a certain percentage of the total variance is explained, typically 80%.

The j-th principal component is computed by: $PC_j = \sum_{q=1}^{m} c_{j,q} \times \Delta r_{T_q}$, where $c_{j,q}, j \in \{1, \dots, m\}$ denotes the j-th PC's coefficients with respect to the yield-to-maturity change with maturity T_q, denoted by Δr_{T_q}. Due to the orthogonality, the yield-to-maturity changes can be expressed as function of the principal components as follows: $\Delta r_{T_q} = \sum_{j=1}^{m} c_{q,j} \times PC_j \approx \sum_{j=1}^{p} c_{q,j} \times PC_j$, where p denotes the number of PCs retained. Consequently, approximate values of the original variables can be computed from the PCs' values, which will be part of the scenarios' definitions. Therefore, almost no information is lost, even though significantly fewer variables are included in Equation (9).

2.4.4. Setting LGD for corporate loans portfolio

When an obligor's asset return falls below $R_{i,CCC-C}$, then the obligor is said to be in default. Consequently, the bank can no longer expect to collect a full repayment of the corporate loan. Whereas Grundke and Pliszka (2017) model the LGD using a beta distribution, I instead apply the STA approach consistently and thus adopt a fixed LGD of 75%[22]. Hence, the new value of the loan of a defaulted group of obligors n (with $n \in \{1, \dots, 346\}$) is computed by $L_{n,default} = L_n \times (1 - LGD) = 0.25 \times L_n$, where L_n denotes the original loan value of obligor group n.

2.4.5. Calculating value of liability side

The bank's liability side consists of deposits of credit institutions, other deposits, and obligations. Following Grundke (2011) and Grundke and Pliszka (2017), it is assumed that the bank remains in its initial rating grade until its default; this is in correspondence with the assumption that the bank's failure is sudden and unexpected. In addition, the possibility of refinancing is not considered. In other words, whichever scenario, the value of the liabilities can be read from the initial balance sheet.

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[21] The trace of a correlation matrix equals $\sum_{j=1}^{p} \lambda_j = p$, with λ_j the eigenvalue of PC_j and p the total number of PCs.. The average proportion of variance explained by one PC equals $\frac{1}{p}$. Hence, when a PC's eigenvalue λ_j is larger than 1, the proportion of variance explained by the j-th PC, denoted by $\frac{\lambda_j}{p}$, is above average.

[22] See Part 3, Title 2, Chapter 3, Section 4, Article 161, EP, 2013.

2.4.6. Capital ratio simulation

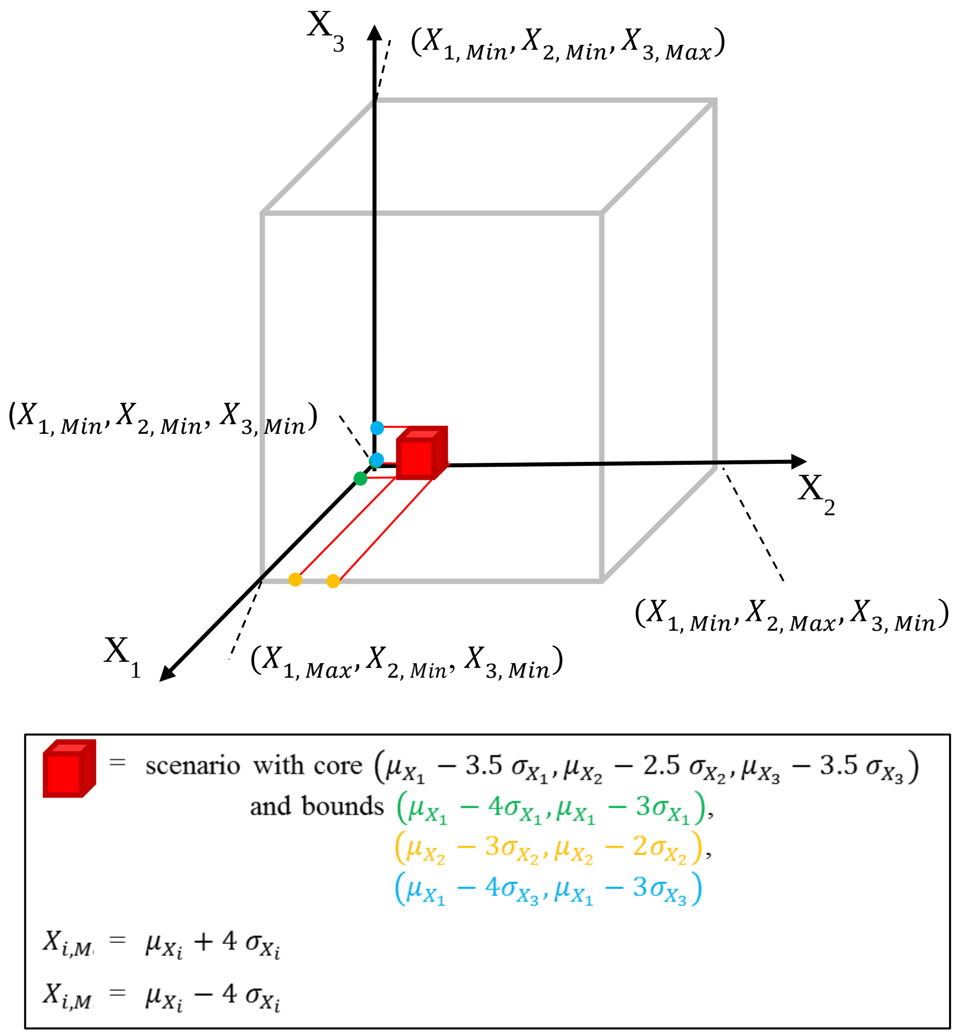


Figure 2.3. Scenario definition in case only three dimensions are considered.

Figure 2.3 shows how scenarios are defined, each for which all balance sheet components and risk weights have to be simulated and included in Equation (2). Whereas the actual scenarios consist of 6 dimensions ($Z, GDPgrowth, PC_1, PC_2, \Delta HPI, \Delta UNEMPL$), only 3 risk factors can be visualized in a figure. The idea remains nevertheless the same for more dimensions. Instead of only considering historical values of each macro-economic risk factor, eight values within the range $[\mu_{X_i} - 4\sigma_{X_i}, \mu_{X_i} + 4\sigma_{X_i}]$[23] are generated for each variable X_i, and combined with every combination of the values of the

[23] This range is expected to include 99.99% of all possible observations of a normally distributed variable. Considering a wide range is important as the identified RSTSs are expected to comprise extreme values for at least some of their dimensions. Although considering an even larger range would result in the identification of a larger number of RSTSs, all additional RSTSs would have a considerably lower probability of occurrence.

remaining five risk factors, resulting in $8^6 = 262144$ scenarios[24]. One such scenario in the 3-dimensional case is presented by the red cube in Figure 2.3. The choice to consider cubes, i.e. intervals of realizations, instead of simple vectors (which would be presented by a dot in Figure 2.3) stems from the fact that the risk factors are continuous variables, and thus the probability of occurrence would otherwise be zero for every scenario. Even though the upper and lower bounds are important for the probability computation (see Section 2.5), the scenarios are defined by the vector that describes the scenario's center point.

For each scenario ω, the values $(z_\omega, gdp_\omega, pc_{1,\omega}, pc_{2,\omega}, \Delta hpi_\omega, \Delta unempl_\omega)$ are inserted in the models described in Sections 2.4.1-2.4.5. The attentive reader will observe that some variability is still incorporated in the computation of the CET1 ratio through inclusion of a random idiosyncratic risk factor, ϵ_n, in the model that predicts the obligors' credit migration (see Section 2.4.2). A Monte Carlo simulation, with 100 repetitions, is executed. In each of these 100 repetitions the credit migration of each of the 346 groups of obligors is simulated.[25] Within each group it is assumed that the obligors are exposed to the same idiosyncratic risk, hence obtain the same asset return in a given scenario and consequently undergo the same credit migration. Once the new ratings are known for all 346 groups, the corresponding risk weights are assigned and the CET1 ratio can be computed. Having obtained the 100 observations of the CET1 ratio for a given scenario, one can draw a distribution of the CET1 ratio, of which the 1%-quantile, denoted by $\mathrm{CET1}_{1\%}$ is subsequently extracted. Note that the difference between $\mathrm{CET1}_{1\%}$ and the current CET1 ratio can be considered as the absolute $\mathrm{VaR}_{99\%}$.[26] If $\mathrm{CET1}_{1\%} < 10.5\%$, the corresponding scenario is identified as a RSTS and should consequently be saved in order to compute its probability.

2.5. Calculating likelihood of the selected scenarios

After having determined all scenarios for which $\mathrm{CET1}_{1\%}$ breaches the lower bound of 10.5%, the most probable scenarios among them have to be identified. As the scenarios are defined as combinations of various risk factor realizations, their respective probabilities can be calculated by analyzing a multivariate probability distribution. The following sections first discuss the marginal distributions of the macro-economic factors, followed by a discussion about the need for a copula

[24] Note that the number of scenarios equals $(the\ number\ of\ values\ considered\ per\ risk\ factor)^6$. Considering 9 values instead of only 8 would lead to more than a doubling of the number of scenarios.

[25] Although a larger number of groups than 346 can be chosen, this considerable increases computation time. The reason is that for each group the asset returns need to be predicted and the according credit migration needs to be simulated. This must be executed 100 times for each of the 262144 scenarios considered.

[26] $Absolute\ VaR_{99\%}(loss) > current\ CET1 - 10.5\% \Leftrightarrow Current\ CET1 - q_{1\%}(CET1) > current\ CET1 - 10.5\% \Leftrightarrow q_{1\%}(CET1) < 10.5\%$.

function to combine these marginal distributions into a multivariate distribution. Note that, besides the multivariate distribution of the stationary transformations of all macro-economic variables, $GDPgrowth$, PC_1, PC_2, ΔHPI and $\Delta UNEMPL$, an additional multivariate distribution for $GDPgrowth$, PC_1 and PC_2 is required in order to draw meaningful samples with the aim to compute the empirical asset return distribution (see Section 2.4.2).

2.5.1. Marginal distribution functions

All macro-economic variables follow a certain empirical marginal probability function, which is approximated by a parametric distribution. For each macro-economic factor two goodness-of-fit tests, the Jarque-Bera test and the Kolmogorov-Smirnov test, are conducted to check whether the distribution can be assumed normal. If this assumption is rejected and the problem lies in the distribution's tails, extreme value theory (EVT) suggests that a generalized pareto distribution (GPD) constitutes a good alternative. The key result of EVT, proven by Gnedenko (1943), is that the tails of many distributions converge to a GPD. Consider the right tail and a threshold u which denotes a certain cutoff point deep into the right tail. Then Equation (11) defines the cumulative distribution in the right tail conditional upon breaching the threshold u (Hull, 2012; McNeil et al., 2005),

$$F_u(y) = P(u < x \leq y + u \,|x > u) = \frac{F(u+y) - F(u)}{1 - F(u)} = G_{\xi,\beta}(y) \quad = \begin{cases} 1 - \left(1 + \xi \dfrac{y}{\beta}\right)^{-\frac{1}{\xi}} & \xi \neq 0 \\ 1 - \exp\left(-\dfrac{y}{\beta}\right) & \xi = 0 \end{cases} \qquad (11)$$

with β the scaling parameter and ξ the shape parameter that determines the heaviness of the tails.

In order to fit the left tail of the distribution, the same approach can be adopted after changing the sign of x, i.e. by working with –x instead. Note that Equation (11) denotes the cumulative probability distribution conditional on exceeding the threshold u. The unconditional distribution, in case $\xi \neq 0$, is provided by Equation (12).

$$F(y) \quad = \begin{cases} \Phi(u^L)\left(1 + \xi \dfrac{|y - u^L|}{\beta}\right)^{-\frac{1}{\xi}} & y < u^L \\ \Phi(y) & u^L \leq y \leq u^R \\ 1 - \left(1 - \Phi(u^R)\right)\left(1 + \xi \dfrac{y - u^R}{\beta}\right)^{-\frac{1}{\xi}} & u^R < y \end{cases} \qquad (12)$$

where u^L (resp. u^R) denotes the threshold in the left (resp. right) tail and where the first (resp. third) component of the equation is only included in case the left (resp. right) tail is modeled by a GPD.[27]

Before estimating the parameters β and ξ, the threshold value u must be chosen. Although fixed rules of thumb are frequently used in practice, such as the 10% rule of DuMouchel (1983) or the rule to set u equal to the 95th percentile (Hull, 2012), they are not applied here since they are not supported by theory (Bommier , 2014; MacDonald and Scarrott, 2012). Instead, the mean residual life (MRL) plots, first introduced by Davison & Smith (1990), are analyzed. An MRL plot shows the mean excesses conditional on exceeding a threshold u, denoted by $E(X-u\,|X>u)$, as a function of u. To determine a correct value for u one can use the threshold stability property of the GPD, which states that if a GPD model with parameters ξ and β_{u_0} is valid for excesses over a threshold u_0, then a GPD model with the same shape parameter ξ, but shifted scale parameter $\beta+\xi(u-u_0)$ is valid for excesses over all thresholds $u>\ u_0$.[28] In other words, for all $u>\ u_0$ it holds that $E(X-u\,|X>u)=\frac{\beta_u}{1-\xi}=\frac{\beta_{u_0}+\xi u}{1-\xi}$, which is a linear function of u. A valid threshold u can consequently be found by searching for the first threshold after which the MRL plot becomes linear (Fawcett, 2012).

Once a value for u is selected, the GPD's parameters β and ξ can be estimated. Although several methods are proposed, the MLE is applied as it is widely used and considered as the most accepted method (see Bommier, 2014 and references therein).

2.5.2. Copulas to obtain the multivariate distribution function

After obtaining the parametrical marginal distributions for $Z, GDPgrowth, PC_1, \dots, PC_p, \Delta HPI$ and $\Delta UNEMPL$, these should be combined into a multivariate distribution function. Simply multiplying each marginal distribution is incorrect, as the assumption of independency implied by this computation is highly unlikely in the case of macro-economic risk factors. Instead, this paper adopts the use of copulas in order to incorporate the dependence of the different variables when constructing the multivariate distribution for $GDPgrowth, PC_1, \dots, PC_p, \Delta HPI$ and $\Delta UNEMPL$. This distribution is subsequently multiplied by $F_1(Z)$, the marginal distribution function of the unobservable systematic risk factor Z, since the latter is assumed independent from the other variables.

[27] Equation (12) is a generalized version of Equations 4.3-4.5 of Grundke and Pliszka (2017).

[28] For a more detailed explanation, see MacDonald and Scarrott (2012) and Bommier (2014).

Sklar's theorem (1959) guarantees that every multivariate distribution of a random vector $(X_1, \dots, X_n)$ with $n \in \mathbb{N}$ can be expressed in terms of its marginal distributions $F_i(x) = \Pr(X_i \leq x)$ and a copula $C: [0,1]^n \rightarrow [0,1]$ such that it holds for all $x_1, \dots, x_n \in \mathbb{R}$ that

$$Pr\big(F_1(X_1) \leq F_1(x_1), \dots, F_n(X_n) \leq F_n(x_n)\big) = F(x_1, \dots, x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big)$$

As described above, a copula function is a mapping of the unit hypercube into the unit interval. The original data must therefore be transformed into data within the range $[0,1]^n$. More specifically, applied on this paper's data, the computation of the joint distribution involves the following steps (Hull, 2012):

1. First, the original variables with marginal distributions $F_2(GDPgrowth)$, $F_3(PC_1)$, $F_4(PC_2)$, $F_5(\Delta HPI)$, $F_6(\Delta UNEMPL)$ are mapped onto variables (A, B, C, D, E) with marginal distribution functions $G_2(A)$, $G_3(B)$, $G_4(C)$, $G_5(D)$ and $G_6(E)$ such that $F_2(GDPgrowth) = G_2(A)$, $F_3(PC_1) = G_3(B)$, $F_4(PC_2) = G_4(C)$, $F_5(\Delta HPI) = G_5(D)$, $F_6(\Delta UNEMPL) = G_6(E)$. Whereas the Gaussian copula assumes that A, B, C, D, E are multivariate normally distributed, the t-copula assumes a multivariate t-distribution.[29]
2. Second, the parameters of the copula are estimated, which imply a certain correlation structure between A, B, C, D and E such that $\big(G_2(A), G_3(B), G_4(C), G_5(D), G_6(E)\big)$ can be translated to $G(A, B, C, D, E)$.
3. Third, the function $G(A, B, C, D, E)$ is mapped onto a multivariate distribution of the originally considered variables, $F(GDPgrowth, PC_1, PC_2, \Delta HPI,\ \Delta UNEMPL)$.

A number of well-known copula functions that define a correlation structure between the original variables are the Gaussian copula, the t-copula, the Clayton copula, the Frank copula and the Gumbel copula.[30] The concrete copulas considered in this paper are the Gaussian copula and the t-copula. Whereas goodness-of-fit-tests have also been conducted on the Clayton, Frank and Gumbel copula, these alternatives are not explained in further detail and no results thereof are presented. The reason is that they do not allow for a correct interpretation in case of a dimension higher than two in combination with negative dependence between the variables (Yan, 2007, p.4), which was exactly

[29] Note that this assumption is made for the marginal distributions of A, B, C, D, E and not for the original variables. This is exactly the main advantage of copula functions: the marginal distribution functions of the original variables can be preserved while defining the correlation structure between them.

[30] The Clayton-, Frank- and Gumbel copula are three examples of the family of Archimedean copulas, which are popular due to possibility to model dependence in high dimensions with a single parameter. Moreover, these four copulas can be described by an explicit formula, whereas this is impossible for the Gaussian copula.

the case in this study's data (see Appendix 5). This paper follows the choice of Grundke and Pliszka (2017) to limit the degrees of freedom of the t-copula to at most 5, as any higher value for the degrees of freedom is considered to result in a t-copula resembling the Gaussian copula. Hence, in total 5 different copula function fits are compared: the Gaussian copula and the t-copula with $df = \{2,3,4,5\}$. Appendix 6 provides a brief description of these copula types. As an in-depth discussion of the Gaussian and t-copula is considered outside the scope of this paper, the interested reader is referred to McNeil et al. (2005), Žežula (2009) and Demarta and McNeil (2004) for more extensive and technical descriptions.

Each of these alternatives is fitted to the data and subsequently evaluated using the Cramér/von Mises test statistic S_T, which is computed as follows:

$$S_T \quad = \quad T \int_{[0,1]^n} \left(C_T(u) - C_{\widehat{\theta_T}}(u)\right)^2 dC_T$$

with $C_T(u)$ denoting the estimated copula function under the H_0, and $C_{\widehat{\theta_T}}(u)$ the empirical copula. Although many goodness-of-fit measures for copulas are proposed, this test statistic is known as delivering more reliable results (Genest, Quessy and Rémillard, 2006; Grundke and Pliszka, 2017). When the copula function under the H_0 is significantly different from the empirical copula, S_T becomes large and the assumption that the copula fits the data will consequently be rejected. Since the probability distribution of S_T under the H_0 is unknown, bootstrapping is required.

As goodness-of-fit tests only answer the question whether a certain model is acceptable to fit the data, another criterion is required to also consider a model's simplicity and to compare the different models. The criteria applied by Grundke and Pliszka (2017) are the Maximum Likelihood (ML), the AIC and the BIC, where the latter two are calculated by Equations (13) and (14), in which k denotes the number of parameters required by the copula function and T denotes the sample size.

$$AIC \quad = \quad -2 \times ML + 2 \times k \tag{13}$$

$$BIC \quad = \quad -2 \times ML + \ln(T) \times k \tag{14}$$

Once the multivariate distribution $F(GDP, PC_1, PC_2, HPI, UNEMPL)$ is obtained, the probability of a scenario's occurrence can be computed by Equation (15).

$$\int_{z_L}^{z_H} \int_{gdp_L}^{gdp_H} \int_{pc_{1,L}}^{pc_{1,H}} \int_{pc_{2,L}}^{pc_{2,H}} \int_{\Delta hpi_L}^{\Delta hpi_H} \int_{\Delta unempl_L}^{\Delta unempl_H} \left(f(z) \times f(gdp, pc_1, pc_2, \Delta hpi, \Delta unempl)\right) dz\, dgdp\, dpc_1\, dpc_2\, d\Delta hpi\, d\Delta unempl \tag{15}$$

where x_L (resp. x_H) denotes the lower (resp. upper) bound of dimension x of the scenario.

To ensure a tractable implementation, the multivariate density distribution, denoted by $f(gdp, pc_1, pc_2, \Delta hpi, \Delta unempl)$ in Equation (15), is expressed in terms of copula functions. The resulting Equation (A.1) is presented in Appendix 7 due to its length. Additionally, the proof of equivalence of both equations is provided in the Appendix 7 as well.

Note that these probability values depend on the grid size in the multidimensional space and can therefore easily be manipulated. This does, however, not pose a significant problem, since the purpose of computing these probabilities is solely to allow for ranking the different RSTSs. Therefore, only the scenarios' relative probability, i.e. as compared to other scenarios' likelihood is required.

2.6. Selection of most different and likely RSTSs

Unlike the approach of Grundke and Pliszka (2017) the framework proposed in this study does not end with the determination of the most likely RSTS. The reason is that solely focusing on the RSTS with the highest probability leaves out important information that, if provided, might have affected decision making of top management when deciding on preventive actions to mitigate extreme risks. Instead, a number of RSTSs are selected that satisfy the following three criteria, which are based upon the criteria set by Kopeliovich et al. (2015):

1. The selection includes the most likely RSTS.
2. The other RSTSs included are among the 10 most likely scenarios as well.
3. The RSTSs selected are maximally different.

Whereas the first two criteria speak for themselves, the third criterion may require some further explanation. As scenarios are defined as vectors of macro-economic factor realizations, the diversity between the RSTSs can be measured by the standardized Euclidean distance between these vectors, as described by Equation (16).

$$d_{i,j} = \sqrt{\frac{(\mathrm{z_i} - \mathrm{z_j})^2}{\sigma_Z^2} + \frac{(\mathrm{gdp_i} - \mathrm{gdp_j})^2}{\sigma_{GDPgrowth}^2} + \frac{(\mathrm{pc_{1,i}} - \mathrm{pc_{1,j}})^2}{\sigma_{PC_1}^2} + \frac{(\mathrm{pc_{2,i}} - \mathrm{pc_{2,j}})^2}{\sigma_{PC_2}^2} + \frac{(\Delta\mathrm{hpi_i} - \Delta\mathrm{hpi_j})^2}{\sigma_{\Delta HPI}^2} + \frac{(\Delta\mathrm{unempl_i} - \Delta\mathrm{unempl_j})^2}{\sigma_{\Delta UNEMPL}^2}} \quad (16)$$

With the aim to satisfy all three criteria, first, the most likely RSTS is kept, second, the 9 next most likely RSTSs are considered and third, 2 of these 9 other RSTSs, i and j, are selected such that the sum of the Euclidean distances, as presented by Equation (17), is maximized.

$$D_{total} = d_{1,i} + d_{i,j} + d_{j,1} \quad (17)$$

3. Implementation of reverse stress test framework

3.1. Data

The data required to model the changes in the balance sheet are described in Table 3.1.

Table 3.1. Data description.

Type of data	Source	Timespan	Freq.
Interest rates Belgian Treasury bills and bonds	Eikon	2006Q4 – 2016Q4	quarterly
Belgian real GDP growth	Eikon	2007Q1 – 2016Q4	quarterly
Belgian HPI	Eurostat	2006Q4 – 2016Q4	quarterly
Belgian unemployment rate	Eikon	2006Q4 – 2016Q4	quarterly
Number of new mortgage defaults in Belgium	CICR	Jan. 2007 – Dec. 2016	monthly
Number of outstanding mortgage loans in Belgium	CICR	Jan. 2007 – Dec. 2016	monthly
Average transition matrix of European corporates	S&P report 2016	1981-2016	/
Default probability corporates of different rating categories	S&P reports 2006-2016	2006-2016	yearly
Number of corporations in each credit rating category	S&P report 2014	2014	/

Yearly default probabilities of mortgage loans can be computed by the historical fraction of defaults:

$$PD_{mortgage}(t) \quad = \quad \frac{number\ of\ new\ defaults\ of\ real\ estate\ morgages\ in\ year\ t}{number\ of\ outstanding\ real\ estate\ mortgages\ in\ year\ t} \qquad (18)$$

As the time series required by Equation (18) only go back to January 2007, they can be considered as the bottleneck in the available data, which consequently leads to limiting the start of the other time series considered to 2007 as well. As the migration and default probability data are not yet reported for 2017, the ends of all time series are limited to 2016.

As opposed to all other data types, the times series of corporate obligors' default probabilities and the average transition matrix over 1981-2016 are based upon European observations instead of only including Belgian companies. Even though Belgian corporations only make up 1.52 % of this European pool of corporates rated by S&P, it is considered an adequate approximation for the portfolio in this paper since about 75% of the latter pool consists of companies located in Western Europe. The economic situations in these countries are strongly intertwined with each other and with the Belgian economy; e.g. if a recession occurs in one of these countries, causing a significant change in the migration matrix, it will also affect the economy and migration probabilities in the other countries. Hence, it is assumed that the migration matrices for Belgium and Europe are the same or similar. A more detailed overview of the distribution of S&P rated corporates over European countries is provided in Appendix 8.

One major data issue encountered is that taking yearly data points would lead to only 10 observations, as default probabilities of mortgage loans are only available from 2007 on and defaults and migration probabilities of corporate loans are reported only until 2016. However, considering a shorter time interval between observations is not straightforward, as corporate default probabilities are only provided on a yearly basis. One must therefore consider the trade-off between the inaccuracy caused by having to disaggregate the yearly PDs into quarterly observed PDs and the inaccuracy of a model based on only 10 data points. Since all other data types are available at least on a quarterly basis, the former approach is expected to have a smaller impact on results. Furthermore, Zhou (2001) provides evidence that applying linear interpolation or other methods to fill in the values of a single lower frequency data series in order to use the information contained in all other higher frequency time series often results in a gain in information.

To accomplish this disaggregation of the yearly PDs into quarterly observations linear interpolation can be applied, i.e. given the default probability in the end of year y, PD_y, the default probability in the i-th quarter of that year can be computed as follows:

$$PD_{y,Q_i} \quad = \quad PD_{y-1} + \frac{PD_y - PD_{y-1}}{4} \times i \tag{19}$$

A second data issue encountered is that the S&P report of 2008 does not include a Europe-specific transition and default matrix. Again, this can be resolved by applying linear interpolation between the migration probabilities of 2007 and 2009.

3.2. Results

The following seven subsections cover the obtained results. Sections 3.2.1-3.2.6 concretize the different model parameters. Afterwards, Section 3.2.7 examines the identified RSTSs.

3.2.1. Dynamic lag model for PD$_{mortgage}$

To obtain a prediction for the default probability of mortgages for the coming period, a model consisting of two parts, denoted by Equations (7) and (8) (see Section 2.4.1), is estimated. The sample data used, which are time series of $PD_{mortgage}$, Belgian GDP growth, unemployment rate and HPI, are plotted in Figure 3.1. Besides the clear non-stationarity of $\ln\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$ and HPI, some persistence can also be seen in $UNEMPL$.

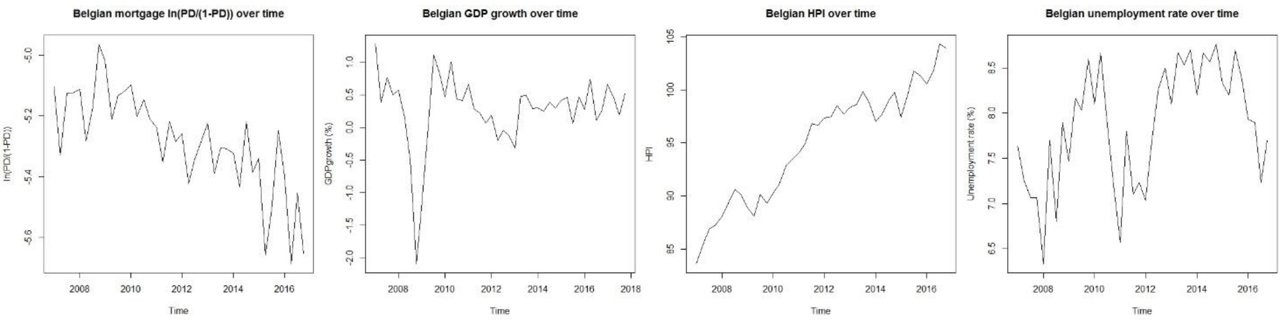


Figure 3.1. $\ln\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$, GDP growth, HPI and unemployment rate over time.

The results of the Augmented Dickey-Fuller (ADF) tests and corresponding required actions are provided in Table 3.2. The ADF tests for $\ln\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$, HPI and $UNEMPL$ confirm the trend-stationarity, resp. non-stationarity of these time series. A deterministic trend term should be included to obtain a statistically correct model to predict the default probability of the mortgage loan portfolio, and the residuals are subsequently regressed on the stationary transformations of the macro-economic factors. The plots of ΔHPI and $\Delta UNEMPL$ and the ADF tests on these transformations, provided in Appendix 10, prove that taking first differences of HPI and $UNEMPL$ is sufficient to obtain stationarity.

Table 3.2. Results of ADF tests.

Variable	Trend?	ADF statistic	p-value	Conclusion	Action
$\ln\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$	Yes	- 6.2435	$5.974*10^{-5}$	Trend-stationary	Include deterministic trend in model
$GDPgrowth$	No	- 2.9657	0.04876	Stationary	/
HPI	Yes	- 2.5441	0.3065	Non-stationary	First differences
$UNEMPL$	No	- 2.2887	0.1812	Non-stationary	First differences

Table 3.3 presents the estimated parameters and criteria of the two parts of the model. The trend term explains 57.09% of the variance in $\ln\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$. The second part of the model is selected as the best model among the 1080 alternatives. Even though other models succeed in explaining a larger part of the residuals' variance, they are much more complex, with many lags of each of the regressors (see Appendix 11). In these cases, the sample size is not sufficient to estimate all coefficients accurately, according to the one-in-ten rule[31]. A trade-off in explanatory power and simplicity is made by both the AIC and the BIC. These information criteria and the one-in-ten rule put forward the model presented in Table 3.3. A significance level of $\alpha = 10\%$ is adopted. The signs of the coefficients corresponding to ΔHPI_t, $\Delta UNEMPL_{t-1}$ and $\Delta UNEMPL_{t-2}$ are consistent with economic intuition and with previous literature (Li, 2014 and references therein). $\Delta UNEMPL_t$, however, turns out to be of negligible importance for the prediction of $\ln\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$. This can be interpreted as follows: an increase in unemployment only positively affects the default probability in the long run. Note that the transformation of $PD_{mortgage}$ into log odds leads to a more complex interpretation of the coefficients. For instance, an increase in ΔHPI_t by one unit is expected to result

[31] The one-in-ten rule advises to have a sample containing at least 10 data observations per explanatory variable (Harrell et al., 1984; Harrell, Lee, Mark, 1996; Peduzzi et al., 1996).

in a decrease of $PD_{mortgage,t}$ of 2.24%.[32] As the p-value of the Q-statistic[33] is large, the assumption that the model's residuals are white noise is not rejected and, hence, the model is concluded to be valid. The model's accuracy has been tested using a simulation that imitates a true out-of-sample forecast by splitting the observations repeatedly into two parts. The resulting Root Mean Squared Percentage Error (RMSPE) equals only 5.31%, which indicates that the model is accurate.

Table 3.3. Parameters and properties of model selected to predict $PD_{mortgage}$.

Model part 1			
Const.	Trend	Adj. R^2	p-value F test
-5.063093****	-0.010665****	0.5709	$1.057*10^{-8}$

Model part 2						
(Lags of) explanatory variables					Adj. R^2	0.3109
Const.	ΔHPI_t	$\Delta UNEMPL_t$	$\Delta UNEMPL_{t-1}$	$\Delta UNEMPL_{t-2}$	AIC	-74.45
0.0198	-0.0227*	-0.0436	0.0789**	0.0516*	BIC	-64.31
					RMSPE	0.0531
					p-val. Q-stat.	0.6533

'****' = sign. on 0.1% level, '***' = sign. on 1% level, '**' = sign. on 5% level, '*' = sign. on 10% level

3.2.2. PCA

Figure A.7 in Appendix 12 shows the Belgian government benchmark interest rates with maturities ranging from 3 months to 30 years over time. Due to the downward trend in all interest rates, non-stationarity is suspected and for most maturities subsequently confirmed by the results of the ADF tests, provided in Table A.8 in Appendix 12. Following Grundke and Pliszka (2017) in taking percentage differences, as expressed by Equation (20), does not offer a solution; the time series of

[32] When $X_1 = k \quad \Rightarrow \quad PD_{mortgage} = e^{\beta_0+\beta_1 k+\beta_2 X_2+\beta_3 X_3+\beta_4 X_4}$

When $X_1 = k+1 \quad \Rightarrow \quad PD_{mortgage}' = e^{\beta_0+\beta_1 (X_1+1)+\beta_2 X_2+\beta_3 X_3+\beta_4 X_4} = e^{\beta_0+\beta_1 X_1+\beta_2 X_2+\beta_3 X_3+\beta_4 X_4} \times e^{\beta_1}$.

$\Rightarrow PD'_{mortgage} - PD_{mortgage} = PD_{mortgage}(e^{\beta_1} - 1)$

[33] The Q-statistic tests whether a model's residuals are coming from a white noise process, which is a sequence of i.i.d. stochastic variables. If this H_0 is rejected, it means that the residuals still contain some persistent correlation structure and the validity of the model considered is rejected.

maturity of 7 years and 9 years are still non-stationary after this transformation. Furthermore, the resulting values would become difficult to interpret due to the frequent occurrence of negative interest rates recently.[34] Therefore, regular changes are computed instead, as expressed by Equation (21). Table A.8 in Appendix 12 additionally confirms that these transformations are stationary for all maturities.

$$\%\Delta r_m(t) = \frac{r_m(t) - r_m(t-1)}{r_m(t-1)} \qquad \forall\, m \in \{3M, 6M, \dots, 30YR\} \tag{20}$$

$$\Delta r_m(t) = r_m(t) - r_m(t-1) \qquad \forall\, m \in \{3M, 6M, \dots, 30YR\} \tag{21}$$

Figures 3.2 and 3.3 are used as visual aids to decide how many PCs should be retained. All three rules of thumb described in Section 2.4.3 (Kaiser-Guttman criterion, Scree plot "elbow" and 80%-rule) consistently lead to the conclusion that two PCs ($PC_1(t), PC_2(t)$) should be retained, together accounting for 90.84% of the total variance of the term structure of risk-free interest rates.

Analyzing Figure 3.4, which presents the factor loadings corresponding to each PC, allows for the interpretation of both PCs. The factor loadings of $PC_1(t)$ all have the same negative sign and this first PC can therefore be interpreted as the average level of the interest rate curve. Whereas the factor loadings of $PC_2(t)$ are also negative for short maturities, they become positive for longer maturities. The second PC can therefore be interpreted as the slope of the interest rate curve.

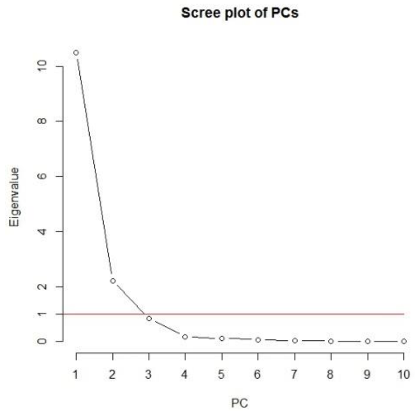


Figure 3.2. Scree plot, denoting each PC's eigenvalue.

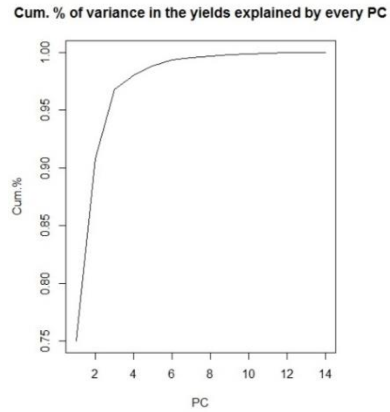


Figure 3.3. Cumulative percentage of variance explained by PCs.

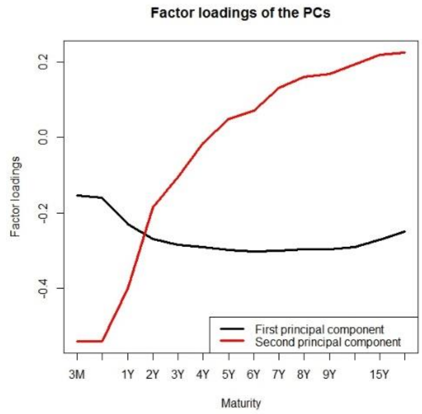


Figure 3.4. Factor loadings of the two PCs retained.

[34] Take for example the case when the interest rate was negative at time t, e.g. -0.1% and becomes positive at time t+1, e.g. 0.1%. The resulting percentage change equals $\frac{0.1\%-(-0.1\%)}{-0.1\%} = -2 < 0$, even though the change was actually positive.

3.2.3. Asset return sensitivity computations

The asset return sensitivities towards $GDPgrowth$, PC_1 and PC_2, estimated using Equation (10) and to be substituted in Equation (9), are presented in Table 3.4. For both the investment grade and the speculative grade, the unobservable systematic risk factor is unimportant. Note that this is beneficial for the interpretability of the identified RSTSs: in the most likely RSTS the value of the unobservable systematic risk factor will always be near zero and the scenarios can therefore be interpreted by only observable risk factors. The sign of $GDPgrowth$ is economically intuitive: an increase in GDP is expected to increase the obligors' returns, ceteris paribus. High interest rates result in expensive loans and are therefore expected to reduce obligors' asset returns. An increase in PC_1 results in a decrease of interest rates over all maturities, consequently encouraging investment. Grundke and Pliszka (2017) explain however that central banks raise (resp. lower) the level of interest rates as a tool to slow down (resp. speed up) the economy in case of large economic growth accompanied by inflation (resp. a recession). The sign of the sensitivity towards PC_1 depends on which of the two reasonings weighs heavier and is hence difficult to predict. This is also shown in Table 3.4, as the sign differs depending on the initial rating grade. An increase in PC_2 results in an increase of the slope of the yields curve. Since a steep yield curve corresponds to a booming economy, the sign of the asset sensitivity towards PC_2 is expected to be positive, which is confirmed in Table 3.4. The RSTSs are therefore expected to consist of a near-zero value for Z and a small value for $GDPgrowth$ and PC_2. The expected value of PC_1 depends on the rating category of the obligors.

Table 3.4. Asset return's sensitivities towards the macro-economic risk factors.

	$Z(t)$	$GDPgrowth(t)$	$PC_1(t)$	$PC_2(t)$
Speculative grade	$9.1357*10^{-77}$	0.9995	0.2888	0.4085
Investment grade	$2.8493*10^{-24}$	0.9150	-0.0776	1.3430

3.2.4. Macro-economic factors' marginal distributions

In order to compute the obligors' asset returns, samples must be drawn from the multivariate distribution $F(GDPgrowth, PC_1, PC_2)$, where the latter can only be obtained after fitting marginal functions and a copula function for the macro-economic risk factors GDPgrowth, PC_1 and PC_2. Additionally, marginal functions for the two remaining variables, $\Delta UNEMPL$ and ΔHPI, and the overall copula function, linking the five factors into a multivariate distribution, are fitted as they will be required in order to compute the RSTS's likelihood.

Table 3.5 shows that, whereas the Kolmogorov-Smirnov (KS) test does not reject the H_0 of normality for any variable, the Jarque-Bera (JB) test rejects this assumption for both $GDPgrowth$ and PC_2. Analyzing the density functions (Figure 3.5) and qqplots (Figure 3.6) allows for the detection of deviations, which appear to be mainly situated in the left tail for $GDPgrowth$ and in the right tail for PC_2. Since the conclusion whether or not to accept the normality assumption varies depending on which test is used, any further analysis is conducted twice, i.e. once using only normal marginal distributions and once using GPD distributions to model the left (resp. right) tail of the distribution of $GDPgrowth$ (resp. PC_2). Analysis of the MRL plots of both variables[35] (Figures 3.7 and 3.8), in combination with ensuring that threshold exceedance occurs at least twice[36] in the sample, results in selecting the following thresholds: $u_{GDPgrowth} = u_L = -1.14$ and $u_{PC_2} = u_R = 2.5$. The scale and shape parameters of the GPD for $GDPgrowth$ (resp. PC_2) are estimated using the MLE and equal $\beta_{GDP} = 1.028576$ and $\xi_{GDP} = -1.077893$ (resp. β_{PC_2}= 2.3452091 and $\xi_{PC_2} = -0.9716526$).

Table 3.5. Results tests for normality of marginal distributions.

	$GDPgrowth$	PC_1	PC_2	ΔHPI	$\Delta UNEMPL$
JB test statistic	79.299	1.4523	25.04	1.8733	2.7755
p-value JB	2.2e-16	0.4838	$3.653*10^{-6}$	0.3919	0.2496
KS test statistic	0.1665	0.14195	0.16397	0.12286	0.10844
p-value KS	0.1943	0.3614	0.2082	0.5411	0.7347

[35] Note that in order to model the left tail of $GDPgrowth$, a change of sign has been implemented for all data observations as the fitting of a GPD distribution in R is always conducted on the right tail.

[36] This is required to estimate the GPD's two parameters, ξ and β.

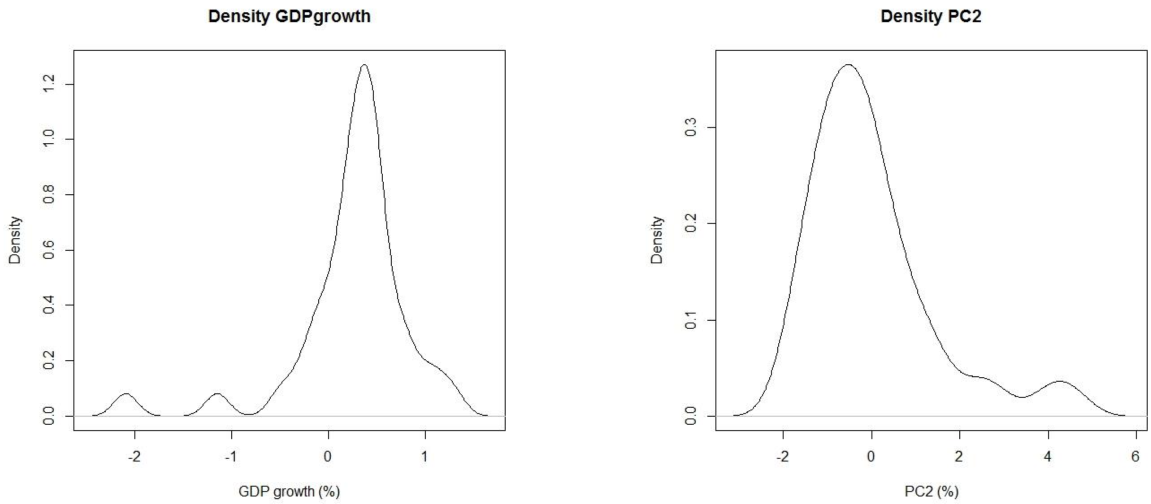


Figure 3.5. Density functions of $GDPgrowth$ and PC_2.

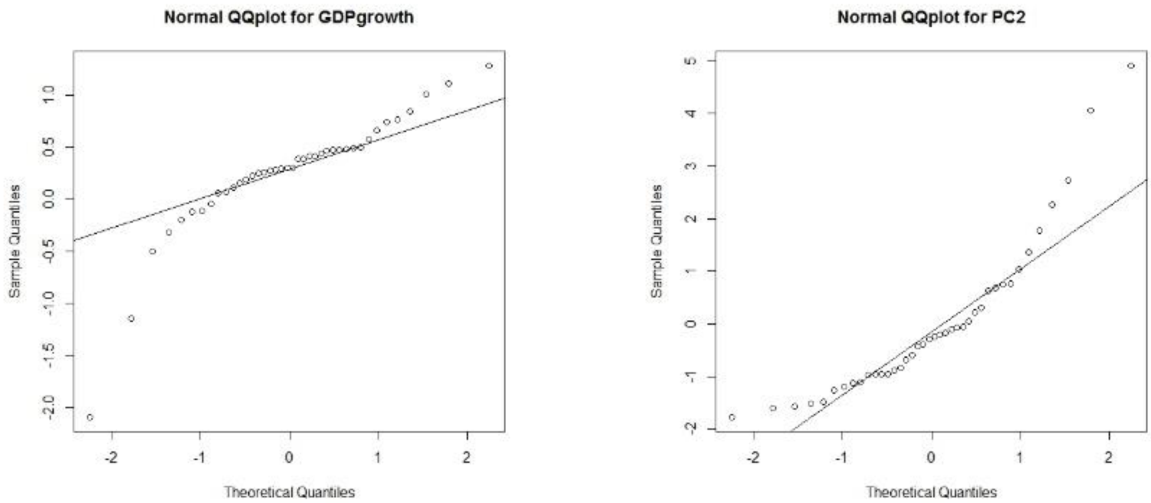


Figure 3.6. QQplots of $GDPgrowth$ and PC_2.

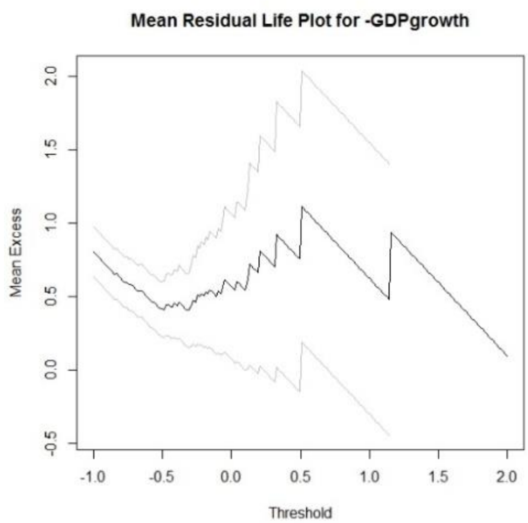


Figure 3.7. MRL plot for $GDPgrowth \times (-1)$.

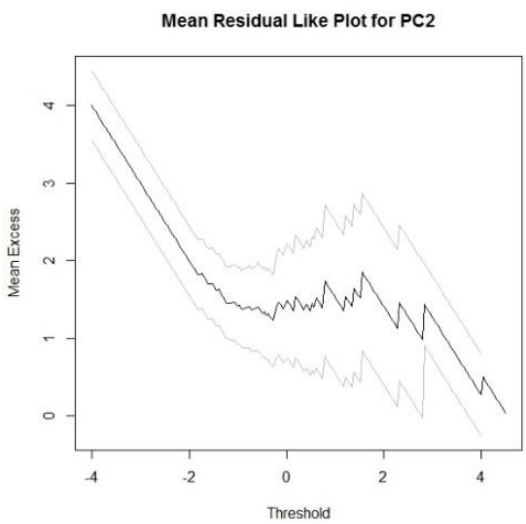


Figure 3.8. MRL plot for PC_2.

3.2.5. Copula goodness of fit

After specifying the macro-economic factors' marginal distributions, they should be combined into a multivariate distribution using a copula function. The value of the Cramér/von Mises test statistic of the different alternatives considered and their corresponding p-values are provided in Table 3.6 (resp. Table 3.7) for the multivariate distribution containing $GDPgrowth$, PC_1 and PC_2 (resp. $GDPgrowth$, PC_1, PC_2, ΔHPI, $\Delta UNEMPL$). The H_0 that the copula constitutes a good fit of the correlation structure is accepted for each alternative. After comparing the different criteria (ML, AIC, BIC) for each alternative copula function, the t-copula with $df = 4$ (respectively the t-copula with $df = 5$) is chosen to model the multivariate distribution of $GDPgrowth$, $PC_1$and PC_2 (respectively $GDPgrowth$, PC_1, PC_2, ΔHPI, $\Delta UNEMPL$). Corresponding parameters to model the correlation structure are provided in Appendix 13.

Table 3.6. Information criteria copula functions for $GDPgrowth$, PC_1 and PC_2.

	Cramér/von Mises test statistic	p-value	ML	AIC	BIC
Normal	0.035429	0.6788	6.265608	-6.531215	-1.4645767
t_{2df}	0.030602	0.6429	6.418042	- 4.836084	1.9194342
t_{3df}	0.034265	0.5789	7.786924	-7.968755	- 0.8183309
t_{4df}	0.036022	0.522	7.984378	- 7.968755	- 1.2132376
t_{5df}	0.036984	0.5	7.935113	- 7.870227	- 1.1147087

Table 3.7. Information criteria copula functions for all five macro-economic factors.

	Cramér/von Mises test statistic	p-value	ML	AIC	BIC
Normal	0.032625	0.532	12.17161	-4.343218	12.545577
t_{2df}	0.029937	0.5789	11.84145	- 1.682892	16.894782
t_{3df}	0.030516	0.5799	15.88734	- 9.774673	8.803001
t_{4df}	0.030921	0.5559	16.72786	- 11.455717	7.121957
t_{5df}	0.031205	0.5699	16.81818	- 11.636352	6.941322

3.2.6. Asset return thresholds

Once the marginal distributions and copula functions have been fitted, samples can be drawn from the macro-economic factors' multivariate distribution and the idiosyncratic risk factor in order to compute the asset return distribution. Table A.5, provided in Appendix 9, presents the cumulative migration probabilities of obligors that are initially of investment or speculative grade. As these numbers were not directly available, they are computed using S&P's average one-year migration matrix over 1981-2016 and the number of corporations in each of the different credit ratings $\{AAA, \dots, CCC - C\}$. A more detailed explanation is additionally provided in Appendix 9.

Table 3.8 presents the asset return thresholds that correspond to the cumulative migration probabilities of Table A.5 for the different settings considered. The result is consistent with intuition, as for a given initial rating grade, the thresholds are in general more dispersed when GPD are considered to model $GDPgrowth$ and PC_2 than in the case when only normal marginal distributions are used as approximations. Additionally, note that initially speculative graded obligors default more easily, as the asset thresholds below which one fails are significantly higher than those for initially investment graded obligors. This is consistent with the reasoning that a larger shock in macro-economic risk factors is required to result in a credit migration of an obligor of investment grade.

Table 3.8. Asset return thresholds.

Initial rating	Marg. distr.	R_{CCC-C}	R_B	R_{BB}	R_{BBB}	R_A	R_{AA}	R_{AAA}
Speculative grade	Normal	- 2.08	- 1.40	0.79	2.92	5.17	7.13	7.13
	GPD	- 2.61	- 1.77	0.89	3.45	5.78	7.55	7.55
Investment grade	Normal	-8.23	-7.69	-6.71	- 4.59	- 0.07	2.25	4.58
	GPD	- 8.85	- 8.23	- 7.19	-4.77	-0.11	2.28	4.92

3.2.7. RSTS determination

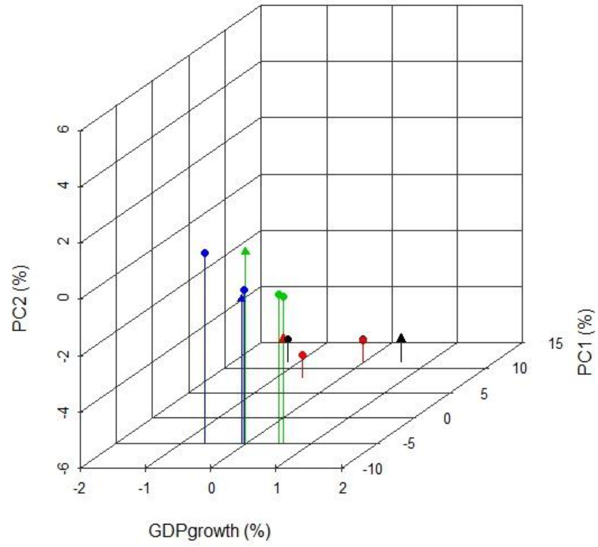
The RSTS determination is performed four times (see Figure 3.9): on the one hand, the bank's corporate loan portfolio is either assumed to be of the investment type or of the speculative type, on the other hand, the marginal distributions for $GDPgrowth$ and PC_2 can be approximated by either normal distributions or GPDs. For the settings with obligors of investment (respectively speculative) grade, the initial CET1 ratio equals 16.74% (respectively 13.76%). For both settings of marginal distributions, the number of RSTSs determined is significantly larger[37] in the case of initially speculative grade obligors than in the case of initially investment grade obligors.

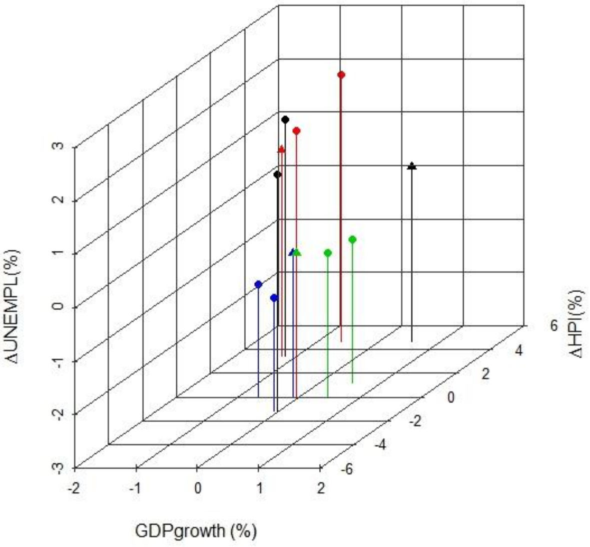
Table 3.9. The four settings for RSTS determination.

	Initial grade of corporate obligors	
Marginal probability distributions	Investment grade, normal distributions	Speculative grade, normal distributions
	Investment grade, normal & GPD	Speculative grade, normal & GPD

For each of the four settings, the 3 'most different RSTSs' out of the 10 most probable RSTSs are visualized in Figure 3.10 and Figure 3.11. Whereas, the triangles represent the most likely RSTSs, the circles denote the two RSTSs that are selected such that the total distance between each of the three RSTSs is maximized. Note that the RSTSs presented are the scenarios which are extreme enough to result in the bank's default, and not necessarily the *most extreme* scenarios. Therefore, the values of some dimensions do not necessarily lead to lower asset returns for obligors or higher default probabilities of mortgage loans on their own. If the other dimensions already ensure the default, the remaining risk factors can be set such to maximize the scenario's probability of occurrence. The relatively high threshold to default (CET1<10.5%) increases the probability of this phenomenon, as it reduces the necessity of all risk factors being extreme.

[37] For specific numbers, see Appendix 14.





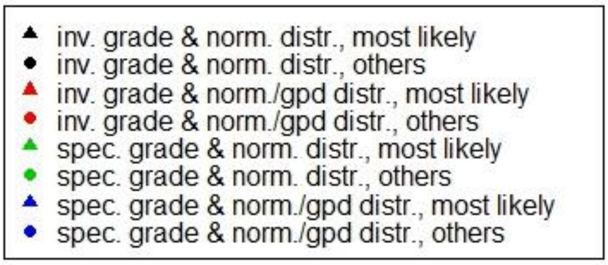


Figure 3.10. Most different RSTSs' values of $GDPgrowth$, PC_1 and PC_2 for four different settings.

Figure 3.11. Most different RSTSs' values of $GDPgrowth$, ΔHPI and $\Delta UNEMPL$ for four different settings.

The first thing to observe when analyzing Figure 3.10 and Figure 3.11 is that none of the six axes represent the unobservable systematic risk factor Z. The reason is that the value of this dimension is close to zero for all RSTSs. This was already predicted before and implies that the systematic unobserved risk factor is virtually irrelevant in the computation of the CET1 ratio. However, its value still plays a role in the computation of the scenarios' probability. Since the unobservable systematic risk factor is assumed to be standard normally distributed, this probability is higher for a lower smaller absolute value of this risk factor. This observation is completely consistent with the results from Grundke and Pliszka (2017).

Second, the black and red (respectively blue and green) RSTSs are clustered together. In other words, results are similar for the settings with the same initial credit grading for the corporate obligors but with different assumptions about the marginal distributions. Only the $GDPgrowth$ values differ slightly between the two settings with speculative grade portfolios; these values are more negative in case PC_2 and $GDPgrowth$ are modeled using GPD distributions (blue) than when only normal marginal distributions are considered (green). This result corresponds to the better modeling of the left tail of $GDPgrowth$. Nevertheless, in general, the differences are small and the results are concluded to be robust for the two different assumptions about the marginal distributions.

Third, note that the results are consistent with the idea that the bank with obligors with an initial speculative grading fails more easily: the RSTSs denoted by the blue and green symbols ~~presented~~ are less extreme, with in general smaller absolute values for all risk factors. This can particularly be observed in Figure 3.10. In general, the $GDPgrowth$ values are lower for the RSTSs of a bank with a portfolio of investment grade, however, not in an extreme sense. The yield curves, obtained by translating PC_1 and PC_2 back to the interest rates over different maturities, are provided in Figure 3.12 for the most likely RSTSs in each of the four settings. Additionally, the mean yield curve over the past 10 years is provided in black. The yield curves are the same for the most likely scenarios for the two settings with an investment grade portfolio, since the values of both PC_1 and PC_2 are the same for these two scenarios. As predicted, the values of PC_1 are positive for the settings with investment grade obligors (i.e. the black and red triangles in Figure 3.10). This is translated to a decrease in the interest rate level over all maturities and, hence, to a downward shift of the yield curve. Furthermore, a large negative value of PC_2 is required for these settings. This represents a tilt of the yield curve, causing it to become inverse. For the settings with a speculative grade portfolio (i.e. the green and blue triangles), the prediction of negative values of PC_1 is confirmed as well. This is translated to an upward shift of the yield curve. The values of PC_2 are not very significant for these settings. Therefore, the corresponding yield curves have approximately the same slope as the mean yield curve. Whereas the $GDPgrowth$ values are not that extreme, the shocks in interest rates are large. Generally, it can be concluded that obligors of speculative grade suffer more from an increase in borrowing cost, whereas obligors of investment grade suffer more from an inverse yield curve, indicating a recession.

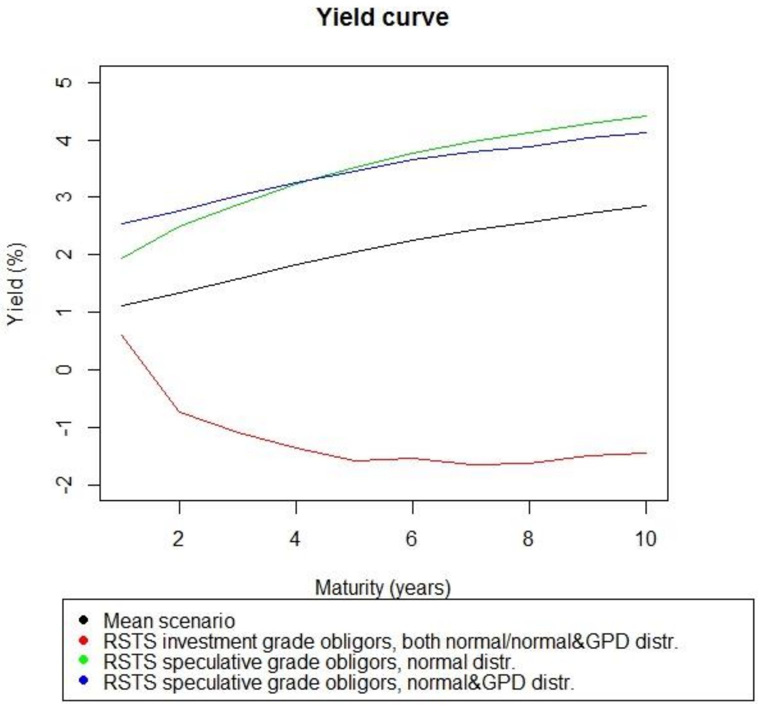


Figure 3.12. Shocks in the yield curve in the most likely RSTS of each of the four settings.

Fourth, whereas most of the values of the growth in GDP and the two PCs correspond to what was predicted, this does not always hold for the change in the HPI and the change in the unemployment rate. Consider for instance the most likely RSTSs in both settings of obligors of investment grade (i.e. the black and red triangles in Figure 3.11). Whereas $\Delta UNEMPL$ does not reach an extreme value, the value of ΔHPI is even positive. Intuitively, this value should be negative, since a downturn in house prices results in a lower residual value of the mortgage loans in case of a default. This result should however be interpreted carefully. It is not the case that a higher value of ΔHPI leads to a higher CET1 ratio. The minimum requirement of 10.5% has also been breached by similar scenarios but with a negative value for the ΔHPI dimension. The CET1 ratio of these similar RSTSs was even lower, but their probability was significantly lower as well. Therefore, these scenarios did not make the top 10 most likely RSTSs and cannot be found in Figures 3.10 and 3.11. The relatively high value of ΔHPI can be logically explained as the house prices in Belgium have steadily grown over the last years, making a sudden downturn considerably unexpected. The scenarios can thus be interpreted as follows: even when house prices increase considerably, the bank can still fail due to unfavorable values for the other risk factors.

Fifth, the higher dispersion of the range of the dimensions $\Delta UNEMPL$ and ΔHPI leads to the conclusion that it are the other three risk factors, $GDPgrowth$, PC_1 and PC_2, that deliver the most important contributions to the bank's default, resulting in more leeway for $\Delta UNEMPL$ and ΔHPI to maximize the scenarios' probability and the Euclidean distance between the RSTSs. This can be explained by the fact that an F-IRB approach is applied for the mortgage loans portfolio, which allows to compute $PD_{mortgage}$ using internal models while keeping the value of $LGD_{mortgage}$ fixed. The $LGD_{mortgage}$ value of only 10% limits the importance of the default probability of mortgage loans and consequently of ΔHPI and $\Delta UNEMPL$. It additionally allows to explain the positive value of ΔHPI, discussed earlier. Additional simulations with $LGD_{mortgage} = 45\%$ have been executed for the two settings with the obligors of investment grade[38]. Whereas no complete analysis of these results is provided here, the main conclusions to be drawn are the following. First, the values for PC_1 and PC_2 are considerably less extreme, the value of $\Delta UNEMPL$ remains the same and the value for ΔHPI decreases significantly, becoming negative for the most likely RSTS. This indicates that the role of the change in house prices in the bank's default, increases. Second, it is observed that, in general, RSTSs are less extreme for this higher $LGD_{mortgage}$ value. This can be explained by the fact that the initial CET1 ratio is only 11.38%, which is already close to the threshold value of 10.5%.

The identified RSTSs can be compared to historical crises. This exercise is executed for the scenarios presented in Figure 3.10 and 3.11 (i.e. with $LGD_{mortgage} = 10\%$). Overall, no great recession is required in order to result in a default of the bank with speculative grade obligors. It is sufficient that economic growth, approximated by $GDPgrowth$, decreases to 0%, while interest rates increase, resulting in loans being more expensive. The scenarios identified for this setting can be considered as a (much) weaker version of the recession in the U.S. in the 1980s. During these years, there was little to no economic growth, no change in unemployment rate and interest rates were set considerably high by the Federal Reserve with the aim to control inflation (Cowan, 1981; Rattner, 1981). A greater recession is required in order to result in a default of the bank with the investment grade portfolio. Especially the downward slope of the yield curve (see Figure 3.12) indicates that the economy is heading towards a recession. Investors expect interest rates to decrease and therefore want to invest in longer-term securities in order to lock in the current higher yields. Additionally, due to the fear of having to reinvest at lower interest rates, investors are less attracted to short-term investments. Hence, demand for long-term (respectively short-term) securities increases (respectively decreases), resulting in higher (respectively lower) prices for securities with long (respectively short) maturities and lower long-term (respectively higher short-term) yields and consequently leading to a

[38] For the settings with the obligors of speculative grade, the initial CET1 ratio of 9.94% already results in immediate default, making further analysis pointless.

flatter or inverted yield curve. Whereas the size of the downward slope of the yield curve can be compared to the one in the U.S. in 1981, the negative interest rates are a typical characteristic of the European financial market during recent years (although the negative interest rates in the RSTSs are still more extreme.). Negative economic growth and an increase in unemployment rate are similar to the situation in the euro area in 2008 (although the GDP growth was even more negative during this crisis) (ECB, 2016). Note that during this financial crisis of 2007-2009, the housing market in Belgium did not slow down significantly. Instead, house prices stagnated. This is consistent with the observation that the ΔHPI dimension of the identified RSTSs does not contribute to the bank's default. In other words, if the bank with the investment portfolio defaults, it is more likely to be caused by an economic downturn, associated with higher unemployment rates and low expectations about future recovery (i.e. inverted yield curve) than by a collapse of the housing market (Delmendo, 2018).

4. Conclusion

The construction of a quantitative reverse stress testing framework is a complex problem, as it needs to account for many potential risks influencing a bank's activities. A limited number of previous research contributions have proposed quantitative RST methodologies. However, none of these models include mortgage loans and none of the models that include corporate obligors have been implemented on European data. The purpose of this study is to reduce this gap by constructing a quantitative RST framework focusing on credit risk from the perspective of a medium to large Belgian savings bank.

The main results of the RST are that for a $LGD_{mortgage}$ of only 10%, shocks in $GDPgrowth$ and the yield curve (represented by PC_1 and PC_2) play the biggest role in the bank's default. The results are consistent with the idea that a bank with obligors of higher credit ratings fail less easily. In other words, in general, larger shocks of $GDPgrowth$, PC_1 and PC_2 are required to result in the bank's default than when its obligors are initially of speculative grade. Whereas speculative grade obligors are more likely to default or downgrade when the yield curve undergoes a large upwards shift, investment grade obligors suffer more from a downward shift combined by an inverse yield curve, indicating a recession. The identified RSTSs can be compared to historical crises. The most likely RSTS for the bank with the speculative portfolio resembles a weaker version of the recession in the U.S. in the early 1980s. For the settings with the investment grade portfolio, the most likely RSTS adopts the yield curve slope of this same recession. In terms of the other risk factors this RSTS resembles the economic situation in the euro area around the financial crisis in 2007-2009, combined with strongly negative interest rates. Increasing $LGD_{mortgage}$ to 45% leads to a lower initial CET1 ratio, already close to the default threshold of 10.5% (and already below 10.5% for the bank with the speculative grade portfolio), and hence, the RSTSs become less extreme. Whereas the relative role of a change in the HPI and unemployment rate, as compared to the other three risk factors, becomes larger in this setting, no large shocks are required to result in the bank's default.

Even though the framework's output satisfies the regulatory requirements, i.e. putting forward concrete scenarios which would result in the bank's bankruptcy and ranking these RSTSs according to probability, these results should nonetheless be interpreted carefully. Two additional remarks are in order. First, the aim of a quantitative RSTs is to determine scenarios relevant for a *given* bank and this paper does not claim to provide a comprehensive framework for *every* possible type of bank. Second, a quantitative RST framework should always be complemented by qualitative RSTs, since some risk types, e.g. reputational risk, are difficult to quantify (FSA, 2009; Grundke and Pliszka, 2017). The framework is flexible in the sense that other risk factors can easily be included in its two main components, i.e. the CreditMetrics™ model for the corporate obligors and the DL model for the mortgage loans. The application of copula functions additionally adds to this flexibility, since it allows for the computation of the identified RSTSs' probability even in the case when the marginal distributions of the individual risk factors are not standard.

As opposed to this strength in flexibility, there are a considerable number of limitations, some of which can be considered as topics to address in further research. First, a more extensive data set is required to ensure the generalizability of the results. The data set used contains only 40 observations, which implies a high estimation risk for both the asset correlations in the CreditMetrics™ model and the model to predict the default probability of mortgage loan[39]. However, whereas the data used in this paper are limited to publically available data sources, it is likely that banks have more accurate data about mortgage loan defaults and migration rates from the corporate loans included in the portfolio at their disposal. Additionally, the further concretization of the stylized balance sheet is possible if the required data are at hand, e.g. a classification of corporates in terms of sector, which allows for the use of sector-specific LGD values. Alternative macro-economic risk factors can also be included.[40] Thanks to the flexibility of copula functions, no strict assumptions are imposed on the distribution of these factors. Hence, although the results provided should be interpreted cautiously due to limited data availability, this paper contributes by proposing a flexible framework which can be used in further research as a procedure in which a bank's available data can be inserted.

Second, this paper defines a bank's health by solely evaluating whether the minimum capital requirements are met. A topic for further research might be to consider other failure definitions, such as a breach of the Net Stable Funding Ratio or the Liquidity Coverage Ratio. Both ratios intend to ensure a bank's liquidity during periods of stressed market conditions, which has been proven important during the financial crisis of 2007-2009.

Third, this framework does not allow to assess the effect of interest rate risk on the bank's own activities[41]. In contrast to the framework proposed by Grundke and Pliszka (2017), it does not apply a net present value approach to compute the values of the different balance sheet components. The reason for this is two-fold. First, this paper focuses on credit risk rather than interest rate risk.

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[39] Note, however, that concerning the selected regression model for $PD_{mortgage}$, the one-in-ten rule of thumb is nevertheless satisfied.

[40] Other risk factors proposed are, for instance, commodity prices or credit spreads (Avouyi-Dovi et al., 2009; Misina et al., 2006, as cited in Grundke and Pliszka, 2017).

[41] Note that interest rate changes are however indirectly included in the CreditMetrics™ model to predict the migration changes of the bank's obligors.

Therefore, it is more intuitive to start from the current value of the components and to compute the new expected values for a given scenario by incorporating the migration changes and default probabilities. Second, the interest rates over the short term are currently negative in Belgium. The resulting discounted values are by many perceived as counterintuitive (EFRAG, 2017; Moreolo, 2016). Interest rates are, however, known to play a significant role in the profitability of banks' maturity transforming activities. Hence, the issue of incorporating this risk type is an intriguing one and could be usefully explored in further research.

Fourth, analogous to Grundke and Pliszka (2017) the balance sheet is assumed to be static, which means that no refinancing operations or issuing of new loans are considered, and that the composition of assets and liabilities thus remains the same. Additionally, the liability side of a bank is assumed to remain unchanged. However, the bank run on DSB Bank in the Netherlands (2009), leading to its bankruptcy, is an example that demonstrates that this is a strong assumption. Further work should therefore shed more light on the effect of sudden deposit withdrawals and refinancing options.

Fifth, all statistical models suffer from model risk and this framework does not differ in that respect. With the aim to reduce this risk, the following measures are taken: First, 1080 models to predict $PD_{mortgage}$ are compared in explanatory power and predicting performance. Second, the RSTSs determination is performed for four different settings instead of only one and the differences in results are discussed in a sensitivity analysis approach. However, model risk remains an issue in the choice of threshold value u. The distribution of the tails of the risk factors only converges to GPD for an u deep in the tails, but the threshold selection is a topic of discussion. Although a sensitivity analysis on the choice of u is out of scope for this study, this could be an area for further research.

A last limitation concerns the computational intensiveness, which is a well-known issue in the RST context. As previously explained, the framework's tractability diminishes when many risk factors are included. The PCA approach already achieves a reduction in scenario dimensionality from 18 to 6, while retaining most of the variance in the interest rate term structure. However, although only 8 steps are considered in each of these 6 dimensions, this results in 262144 scenarios, for each of which a Monte Carlo simulation needs to be conducted in order to compute the absolute VaR of the CET1 ratio conditional upon that scenario. Areas for further research consist therefore of the implementation of other dimensionality reduction methods and smarter search algorithms instead of the simple grid search.

5. References

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Appendix

Appendix 1

Figure A.1 represents the FSA's integrated stress testing framework (FSA, 2009). Two types of analyses can be distinguished. On the one hand, there is the micro-prudential analysis, covering both the bank's own stress testing and the stress tests conducted by the supervisory institution. On the other hand, a macro-prudential analysis comprises the conduct of a system-wide stress test. The EBA guidelines are structured in a similar way (EBA, 2015). Note that reverse stress testing is considered a part of the firms' own stress testing.

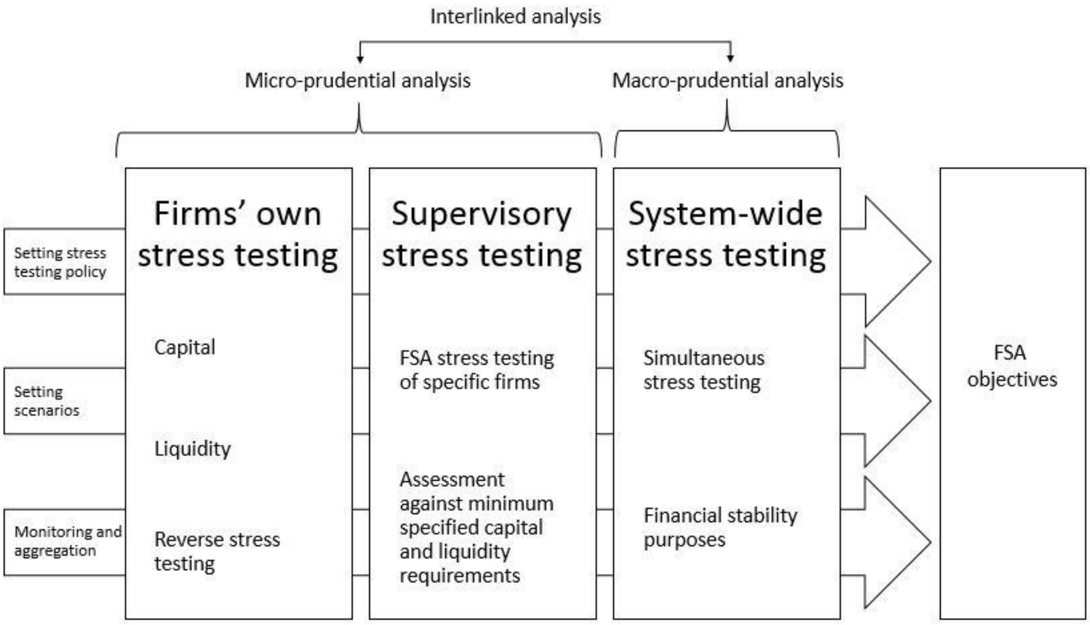


Figure A.1. FSA's integrated stress testing framework.
(Retrieved from: FSA, 2009)

Appendix 2

The Basel Accords are a set of recommendations on regulation in the banking sector, issued by the BCBS. The recommendations are not enforced directly, but are instead translated to national laws to which the banks in the corresponding country should comply. So far, three Basel Accords are published.

Basel I is published in 1988 and focuses solely on credit risk and capital requirements. Assets were classified into five groups, each of which correspond to a different degree of credit risk and are assigned a different risk weight (between 0% and 100%). A minimum capital ratio of 8% was imposed. An amendment was made in 1996 in order to include market risk. More specifically, banks should also consider potential trading losses resulting from changes in market prices.

Basel II, first issued in 2004 and frequently updated in the years thereafter, served as a response to the criticism that Basel I was not sufficiently risk sensitive. The Basel II framework contain three pillars. Pillar 1 covers the minimum capital requirements. Besides credit and market risk, operational risk is additionally considered. Instead of the simple asset classification under Basel I, banks can choose between three methods to compute the credit risk component: the Standard approach (STA), the Foundation Internal Rating approach (F-IRB) and the Advanced Internal Rating approach (A-IRB). The STA approach is similar to the method under Basel I. Asset classification is based on external ratings from recognized rating agencies and in order to increase risk sensitivity, a broader range of risk weights is provided. Under the F-IRB approach banks can apply internal rating systems in order to compute the counterparties' default probability. Risk weighted assets are computed as a function of LGD and EAD, which remain defined by regulatory authorities. Under the A-IRB approach also LGD and EAD are computed internally by the bank itself, although certain minimum requirements should be satisfied. Basel II additionally distinguishes different capital types: core capital, Tier 1 capital, Tier 2 capital and Tier 3 capital, of which the latter is removed in Basel III. The key distinguishing characteristics between the different types are their reliability and the loss absorption capacity. A comprehensive overview of the different components included in every capital type is out of the scope of this paper, but can be found in the CRR, Part 2 (EP, 2013). The minimum requirements for each capital type are presented in Table 2.2. Pillar 2 covers supervisory review and constitutes a framework for handling risk types not covered by pillar 1, such as systemic-, concentration-, reputational- and liquidity risk. Pillar 3 deals with market discipline. Its aim is to increase transparency of the capital structure and risk profile through improved disclosure.

Although Basel III was agreed upon by the members of the BCBS in 2011, its implementation is extended to 2019. The adjustments to the Basel framework aim to address the deficiencies in banking regulation revealed by the financial crisis in 2007-2009. It introduces a minimum leverage ratio and two liquidity ratios: the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). Additionally, it imposes the phasing-in of higher minimum capital requirements and conservation buffers.

The term 'Basel IV' refers to the changes to these Basel Accords in 2016 and 2017 but is not recognized by regulators. These changes limit the potential reduction in capital required resulting from applying the IRB approach as compared to the STA. More specifically, a floor is set of 72.5% of the capital requirement that would be imposed under the STA (PwC, 2017).

Appendix 3

Consider the example in which one wants to regress ln $\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$ on Belgian HPI. Figures A.2 and A.3 show that both time series are non-stationary. A simple linear regression model would lead to an $R^2 = 52.73\%$. The conclusion that HPI explains 52.73% of the variance in ln $\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$ is incorrect, as this high value for the R^2 is largely explained by the common underlying driver, time.

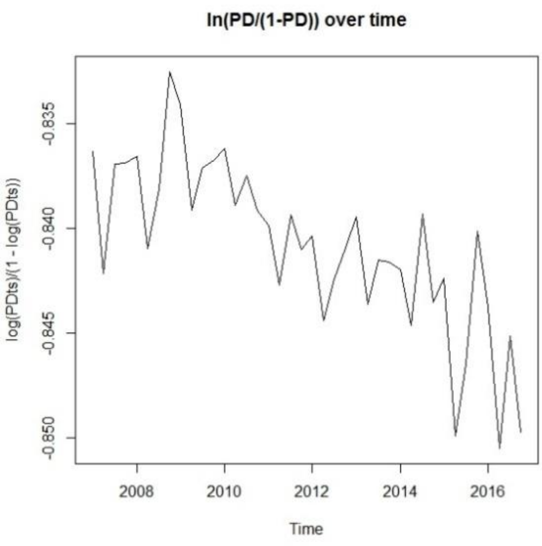


Figure A.2. Belgian $ln\left(\frac{PD_{mortgage}}{1-PD_{mortgage}}\right)$ over time.

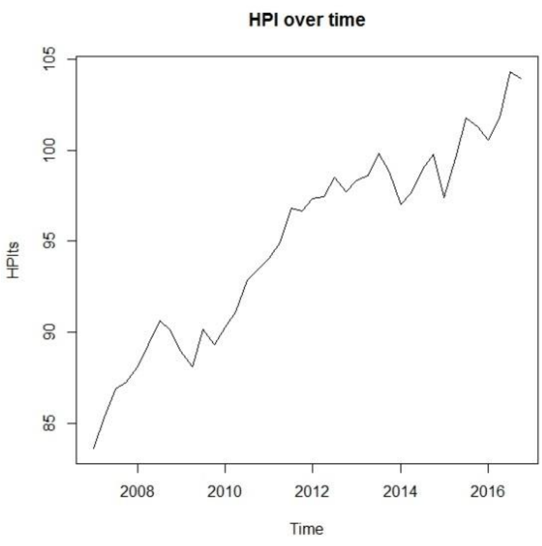


Figure A.3. Belgian HPI over time.

In order to decide how the analysis should be correctly executed, the procedure outlined in Figure A.4 can be followed.

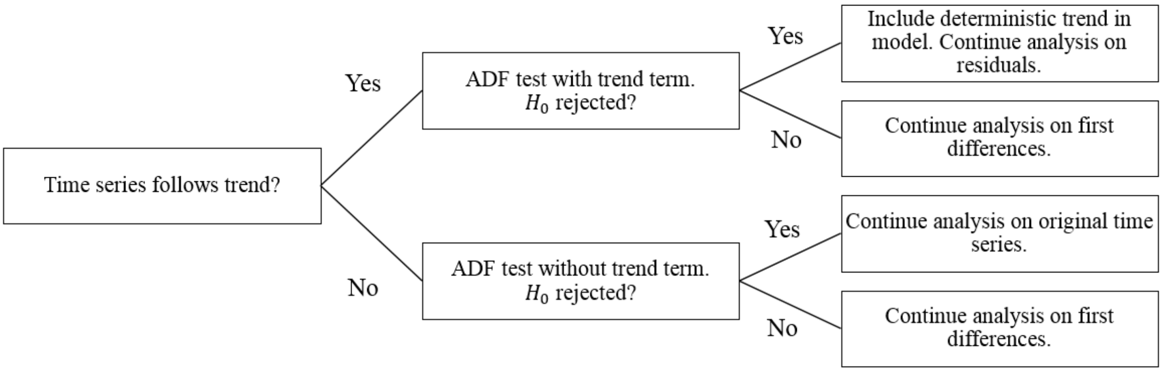


Figure A.4. Procedure for modeling time series. (Adapted from: Croux, 2017)

The procedure starts with the performance of an ADF test in order to test the H_0 of non-stationarity (also referred to as 'unit root') or stochastic trend. In the case when the time series follows a clear trend, a trend term is included in the test equation. The null hypothesis (H_0) states that the deviations from this linear trend follow a random walk without drift and thus that the time series follows a stochastic trend. If the H_0 is rejected, one should include a deterministic trend in the model and continue analysis on the residuals of this model. On the other hand, if the H_0 is not rejected, the time series is concluded to be non-stationary and one should continue further analysis on the first differences. When the time series does not follow a clear trend, no trend term should be included in the test equation. The corresponding H_0 assumes non-stationarity. If this hypothesis is rejected, one can continue the analysis with the original time series. Otherwise, one should again take first differences.

Appendix 4

Demey et al. (2004) explain how an asymptotic maximum likelihood estimator (MLE) approach can be adopted to approximate the binomial MLE, which is often used to estimate asset correlations. However, while the binomial MLE considered by Demey et al. (2004) maximizes the likelihood of the number of defaults in each category at a fixed point in time, the MLE required in this paper should maximize the likelihood of the number of defaults in a fixed credit rating category within a range of time periods. Hence, whereas Demey et al. (2004) sum the likelihoods conditional upon a category over the different categories, this study sums, for each credit rating category separately, the likelihoods conditional upon a time period over the different time periods. Demey et al. (2004) explain that in order to approximate the binomial shaped log-likelihood by an asymptotic MLE, it must hold that $PD_{category} = \frac{number\ of\ defaults\ in\ category}{number\ of\ companies\ in\ category}$ for every category. As this study assumes that the asset correlations may differ for different credit ratings, it is required to maximize the asymptotic MLE separately for each credit rating. Hence the translation of the requirement of Demey et al. (2004) is the following:

$$PD_{i,t} = \frac{number\ of\ corporations\ of\ rating\ i\ defaulted\ at\ period\ t}{number\ of\ corporations\ of\ rating\ i\ at\ period\ t}$$

Additionally, as the correctness of the asymptotic MLE relies on the law of large numbers, it is required that the number of corporations in each rating grade is high at each point in time. As the latter requirement may not be met by all credit ratings (e.g. CCC-C), the different ratings are pooled into two larger categories, 'investment grade' (AAA - BBB) and 'speculative grade' (BB - C).

The equivalence between the log-likelihood estimator with binomial shape applied by Grundke and Pliszka (2017) and Equation (10) needs to be proven. The formula used in Grundke and Pliszka (2017) is given below.

$$l_i = \sum_{t=1}^{T} \ln \int_{-\infty}^{+\infty} \binom{N_i(t)}{d_i(t)} \times q_i\big(z, GDPgrowth(t), PC_1(t), PC_2(t)\big)^{d_i(t)} \times$$

$$\Big(1 - q_i\big(z, GDPgrowth(t), PC_1(t), PC_2(t)\big)\Big)^{N_i(t)-d_i(t)} \times \phi(z)dz$$

The main part of the integrand, everything except $\phi(z)$, denotes the probability of finding $d = d_i(t)$ in a Binomial distribution $d \sim B(N_i, q_i)$.

Now consider the case for large N_i, for which the Binomial distribution becomes approximately normal:

$$d \sim N(q_i N_i, N_i q_i (1 - q_i))$$

The probability of finding $d = d_i(t)$ can then be approximated as:

$$\Phi\left(\frac{d_i(t) + 1 - q_i N_i}{N_i q_i (1 - q_i)}\right) - \Phi\left(\frac{d_i(t) - q_i N_i}{N_i q_i (1 - q_i)}\right)$$

However, since the relevant values of d increase with N_i, it is better to define a new variable:

$$\mu = \frac{d}{N_i} \sim N(q_i, \frac{q_i(1 - q_i)}{N_i})$$

The probability of finding $d = d_i(t)$ is equal to the probability of finding $\mu = \mu_i(t) = \frac{d_i(t)}{N_i}$, hence:

$$\Phi\left(N_i\frac{\mu_i(t)+\frac{1}{N_i}-q_i}{q_i(1-q_i)}\right)-\Phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)=\Phi\left(N_i\frac{\mu_i(t)+d\mu_i-q_i}{q_i(1-q_i)}\right)-\Phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)$$

$$\approx\frac{d}{d\mu_i}\left(\Phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)\right)d\mu_i$$

In the first step $\frac{1}{N_i}$ is rewritten as $d\mu_i$ to denote that it becomes infinitesimally small for $N_i\rightarrow\infty$ and that it can be grouped with $\mu_i(t)$. In the second step the difference of the two cumulative distribution functions is approximated as the derivative multiplied with the interval. This is again valid because the interval is small for large N_i. Note that one cannot substitute this derivative by the marginal distribution function, $\phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)$, because the derivative is to μ_i, not to the whole of $N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}$.

The starting formula can thus be rewritten as:

$$l_i=\sum_{t=1}^{T}\ln\int_{-\infty}^{+\infty}\frac{d}{d\mu_i}\left(\Phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)\right)d\mu_i\times\phi(z)dz$$

Note that the integration is still over z, not over μ_i. Moreover, $d\mu_i$ is merely a constant factor equal to $\frac{1}{N_i}$. It can therefore be brought outside of the integral and even be neglected as it does not influence the remainder of the derivation.

$$=\sum_{t=1}^{T}\ln\int_{-\infty}^{+\infty}\frac{d}{d\mu_i}\left(\Phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)\right)\times\phi(z)dz$$

$$=\sum_{t=1}^{T}\ln\frac{d}{d\mu_i}\left(\int_{-\infty}^{+\infty}\Phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)\times\phi(z)dz\right)$$

Note that, since N_i is considered large, $\Phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)$ approximates a step function:

$$\Phi\left(N_i\frac{\mu_i(t)-q_i}{q_i(1-q_i)}\right)\approx\begin{cases}1\ if\ q_i\leq\mu_i\\0\ if\ q_i\geq\ \mu_i\end{cases}$$

The integral can therefore be rewritten as the integral of $1\times\phi(z)$ over all z for which $q_i\leq\mu_i$. The threshold value, z_{thresh}, is the value of z for which $q_i=\mu_i$.

$$q_i(z_{thresh},GDP,PC_1,PC_2)=\Phi\left(\frac{R_{i,8}-\sqrt{\rho_{i,Z}}\,z_{thresh}-\rho_{i,GDP}GDP(t)-\ \sum_{j=1}^{p}\rho_{i,PC_j}pc_j(t)}{\sqrt{1-\rho_{i,Z}}}\right)=\mu_i$$

$$\rightarrow\frac{R_{i,8}-\sqrt{\rho_{i,Z}}\,z_{thresh}-\rho_{i,GDP}GDP(t)-\sum_{j=1}^{p}\rho_{i,PC_j}pc_j(t)}{\sqrt{1-\rho_{i,Z}}}=\Phi^{-1}(\mu_i)$$

$$\rightarrow z_{thresh} = \frac{R_{i,8} - \rho_{i,GDP} GDP(t) - \sum_{j=1}^{p} \rho_{i,PC_j} pc_j(t) - \Phi^{-1}(\mu_i)\sqrt{1-\rho_{i,Z}}}{\sqrt{\rho_{i,Z}}}$$

Since $\frac{dq_i}{dz} < 0$, this means that: $\begin{cases} q_i \leq \mu_i \text{ } if \text{ } z \geq z_{thresh} \\ q_i \geq \mu_i \text{ } if \text{ } z \leq z_{thresh} \end{cases}$.

Hence:

$$l_i = \sum_{t=1}^{T} \ln \frac{d}{d\mu_i} \left(\int_{z_{thresh}}^{+\infty} \phi(z) dz \right)$$

$$= \sum_{t=1}^{T} \ln \frac{d}{d\mu_i} \left(1 - \Phi(z_{thresh})\right)$$

$$= \sum_{t=1}^{T} \ln \left(-\phi(z_{thresh}) \frac{dz_{thresh}}{d\mu_i} \right)$$

$$= \sum_{t=1}^{T} \ln \frac{\sqrt{1-\rho_{i,Z}}}{\sqrt{\rho_{i,Z}}} \phi(z_{thresh}) \frac{d\Phi^{-1}(\mu_i)}{d\mu_i}$$

$$= \sum_{t=1}^{T} \ln \frac{\sqrt{1-\rho_{i,Z}}}{\sqrt{\rho_{i,Z}}} \frac{\phi(z_{thresh})}{\phi(\Phi^{-1}(\mu_i))}$$

■

Appendix 5

Table A.1. Correlation matrix of the macro-economic risk factors.

	$GDPgrowth$	PC_1	PC_2	ΔHPI	$\Delta UNEMPL$
$GDPgrowth$	1	- 0.4112	-0.6051	0.3024	-0.0721
PC_1		1	$7.2958*10^{-16}$	-0.1291	0.3979
PC_2			1	-0.1374	-0.0616
ΔHPI				1	0.0913
$\Delta UNEMPL$					1

Appendix 6

The Gaussian copula is the best-known copula type and can be applied in a large number of contexts, since it allows for any marginal distributions, combined with any pairwise dependence correlation matrix. However, a drawback is that by considering only pairwise correlations between the different variables, the Gaussian copula may overlook part of the dependence structure.

The t-copula can be considered as a representation of the dependence structure in a multivariate t-distribution (Demarta and McNeil, 2004). This copula type is frequently applied, particularly in the context of modeling multivariate financial return data. Previous research (Breymann et al., 2003; Mashal and Zeevi, 2002) has demonstrated that the empirical fit of the t-copula often considerably exceeds that of the Gaussian copula. The latter can be explained by the better ability of the t-copula to capture dependency in extreme values of the marginal distributions, which is relevant in the context of (reverse) stress testing. More specifically, changing the degrees of freedom parameter allows to set the degree of tail dependency (Dorey and Joubert, 2005).

Appendix 7

The probability of a certain scenario's occurrence $(z, gdp, pc_1, pc_2, hpi, unempl)$ is computed by the following formula, which is an adjusted version of a formula in Grundke and Pliszka (2013) so that it can be applied to the variables considered in this paper:

$$P\big(\mathrm{z_L} < \mathrm{Z} \le \mathrm{z_H}, \mathrm{gdp_L} < \mathrm{GDP} \le \mathrm{gdp_H}, \mathrm{pc_{1,L}} < \mathrm{PC_1} \le \mathrm{pc_{1,H}}, \mathrm{pc_{2,L}} < \mathrm{PC_2} \le \mathrm{pc_{2,H}}, \mathrm{hpi_L} < \mathrm{HPI} \le \mathrm{hpi_H}, \mathrm{unempl_L} < \mathrm{UNEMPL} \le \mathrm{unempl_H}\big)$$

$$= \int_{\mathrm{z_L}}^{\mathrm{z_H}} \int_{\mathrm{gdp_L}}^{\mathrm{gdp_H}} \int_{\mathrm{pc_{1,L}}}^{\mathrm{pc_{1,H}}} \int_{\mathrm{pc_{2,L}}}^{\mathrm{pc_{2,H}}} \int_{\mathrm{hpi_L}}^{\mathrm{hpi_H}} \int_{\mathrm{unempl_L}}^{\mathrm{unempl_H}} \mathrm{f(z, gdp, pc_1, pc_2, hpi, unempl) dz\, dgdp\, dpc_1 dpc_2 dhpi\, dunempl}$$

$$= (\mathrm{F_1(z_H) - F_1(z_L)}) \times$$

$$(\mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,H}), F_4(pc_{2,H}), F_5(hpi_H), F_6(unempl_H)}\big) - \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,H}), F_4(pc_{2,H}), F_5(hpi_H), F_6(unempl_L)}\big)$$

$$- \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,H}), F_4(pc_{2,H}), F_5(hpi_L), F_6(unempl_H)}\big) + \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,H}), F_4(pc_{2,H}), F_5(hpi_L), F_6(unempl_L)}\big)$$

$$- \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,H}), F_4(pc_{2,L}), F_5(hpi_H), F_6(unempl_H)}\big) + \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,H}), F_4(pc_{2,L}), F_5(hpi_H), F_6(unempl_L)}\big)$$

$$+ \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,H}), F_4(pc_{2,L}), F_5(hpi_L), F_6(unempl_H)}\big) - \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,H}), F_4(pc_{2,L}), F_5(hpi_L), F_6(unempl_L)}\big)$$

$$- \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,L}), F_4(pc_{2,H}), F_5(hpi_H), F_6(unempl_H)}\big) + \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,L}), F_4(pc_{2,H}), F_5(hpi_H), F_6(unempl_L)}\big)$$

$$+ \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,L}), F_4(pc_{2,H}), F_5(hpi_L), F_6(unempl_H)}\big) - \mathrm{C}\big(\mathrm{F_2(gdp_H), F_3(pc_{1,L}), F_4(pc_{2,H}), F_5(hpi_L), F_6(unempl_L)}\big)$$

$$+ C\left(F_2(gdp_H), F_3(pc_{1,L}), F_4(pc_{2,L}), F_5(hpi_H), F_6(unempl_H)\right) - C\left(F_2(gdp_H), F_3(pc_{1,L}), F_4(pc_{2,L}), F_5(hpi_H), F_6(unempl_L)\right)$$

$$+ C\left(F_2(gdp_H), F_3(pc_{1,L}), F_4(pc_{2,L}), F_5(hpi_L), F_6(unempl_L)\right) - C\left(F_2(gdp_L), F_3(pc_{1,H}), F_4(pc_{2,H}), F_5(hpi_H), F_6(unempl_H)\right)$$

$$+ C\left(F_2(gdp_L), F_3(pc_{1,H}), F_4(pc_{2,H}), F_5(hpi_H), F_6(unempl_L)\right) + C\left(F_2(gdp_L), F_3(pc_{1,H}), F_4(pc_{2,H}), F_5(hpi_L), F_6(unempl_H)\right)$$

$$- C\left(F_2(gdp_L), F_3(pc_{1,H}), F_4(pc_{2,H}), F_5(hpi_L), F_6(unempl_L)\right) + C\left(F_2(gdp_L), F_3(pc_{1,H}), F_4(pc_{2,L}), F_5(hpi_H), F_6(unempl_H)\right)$$

$$- C\left(F_2(gdp_L), F_3(pc_{1,H}), F_4(pc_{2,L}), F_5(hpi_H), F_6(unempl_L)\right) - C\left(F_2(gdp_L), F_3(pc_{1,H}), F_4(pc_{2,L}), F_5(hpi_L), F_6(unempl_H)\right)$$

$$+ C\left(F_2(gdp_L), F_3(pc_{1,H}), F_4(pc_{2,L}), F_5(hpi_L), F_6(unempl_L)\right) + C\left(F_2(gdp_L), F_3(pc_{1,L}), F_4(pc_{2,H}), F_5(hpi_H), F_6(unempl_H)\right)$$

$$- C\left(F_2(gdp_L), F_3(pc_{1,L}), F_4(pc_{2,H}), F_5(hpi_H), F_6(unempl_L)\right) - C\left(F_2(gdp_L), F_3(pc_{1,L}), F_4(pc_{2,H}), F_5(hpi_L), F_6(unempl_H)\right)$$

$$+ C\left(F_2(gdp_L), F_3(pc_{1,L}), F_4(pc_{2,H}), F_5(hpi_L), F_6(unempl_L)\right) - C\left(F_2(gdp_L), F_3(pc_{1,L}), F_4(pc_{2,L}), F_5(hpi_H), F_6(unempl_H)\right)$$

$$+ C\left(F_2(gdp_L), F_3(pc_{1,L}), F_4(pc_{2,L}), F_5(hpi_H), F_6(unempl_L)\right) + C(F_2(gdp_L), F_3(pc_{1,L}), F_4(pc_{2,L}), F_5(hpi_L), F_6(unempl_H))$$

$$- C(F_2(gdp_L), F_3(pc_{1,L}), F_4(pc_{2,L}), F_5(hpi_L), F_6(unempl_L))) \qquad (A.1)$$

In this formula,

- $f(z, gdp, pc_1, pc_2, hpi, unempl)$ denotes the multivariate density function,
- $F_1(z), F_2(gdp), F_3(pc_1), F_4(pc_2), F_5(hpi), F_6(unempl)$ are the marginal distribution functions of the macro-economic factors, $Z, GDPgrowth, PC_1, PC_2, \Delta HPI$ and ΔUNEMPL, evaluated at z, gdp, pc_1, pc_2, hpi and $unempl$ respectively.
- $C(.)$ denotes the applied copula function
- $\left(z_H, gdp_H, c_{1,H}, c_{2,H}, hpi_H, unempl_H\right)$ $(resp. \left(z_L, gdp_L, c_{1,L}, c_{2,L}, hpi_L, unempl_L\right))$ are the upper bounds of the scenario $(z, gdp, pc_1, pc_2, hpi, unempl)$ of which the probability of occurrence is computed. These upper bounds are computed as follows:

$$\begin{pmatrix} z_H & gdp_H & c_{1,H} & c_{2,H} & hpi_H & unempl_H \\ z_L & gdp_L & c_{1,L} & c_{2,L} & hpi_L & unempl_L \end{pmatrix} = \begin{pmatrix} z & gdp & c_1 & c_2 & hpi & unempl \\ z & gdp & c_1 & c_2 & hpi & unempl \end{pmatrix} \pm 0.5 \times factor\ specific\ step\ size.$$

Note that realizations of $GDPgrowth$ and ΔHPI and $\Delta UNEMPL$ are denoted by $gdp, hpi, unempl$ for clarity.

The proof of this formula is provided below. For clarity the variables $z, gdp, pc_1, pc_2, hpi, unempl$ are replaced by $x_1, x_2, x_3, x_4, x_5, x_6$.

$$\int_{x_1^L}^{x_1^H}\int_{x_2^L}^{x_2^H}\int_{x_3^L}^{x_3^H}\int_{x_4^L}^{x_4^H}\int_{x_5^L}^{x_5^H}\int_{x_6^L}^{x_6^H} f(x_1,x_2,x_3,x_4,x_5,x_6)dx_1\,dx_2\,dx_3\,dx_4\,dx_5\,dx_6$$

x_1 *is independent from the other variables.*

$$= \left(F_1(x_1^H)-F_1(x_1^L)\right)\times\int_{x_2^L}^{x_2^H}\int_{x_3^L}^{x_3^H}\int_{x_4^L}^{x_4^H}\int_{x_5^L}^{x_5^H}\int_{x_6^L}^{x_6^H} f(x_2,x_3,x_4,x_5,x_6)\,dx_2\,dx_3\,dx_4\,dx_5dx_6$$

$$= \left(F_1(x_1^H)-F_1(x_1^L)\right)$$

$$\times\int_{x_2^L}^{x_2^H}\int_{x_3^L}^{x_3^H}\int_{x_4^L}^{x_4^H}\int_{x_5^L}^{x_5^H}\int_{x_6^L}^{x_6^H} f_2(x_2)f_3(x_3)f_4(x_4)f_5(x_5)f_6(x_6)c\left(F_2(x_2),F_3(x_3),F_4(x_4),F_5(x_5),F_6(x_6)\right)dx_2\,dx_3\,dx_4\,dx_5dx_6$$

$$A = \left(F_1(x_1^H)-F_1(x_1^L)\right)$$

$$\int_{x^L}^{x_H} f'(x)dx = \int_{f(x^L)}^{f(x_H)} df(x)$$

$$= A$$

$$\times\int_{F_2(x_2^L)}^{F_2(x_2^H)}\int_{F_3(x_3^L)}^{F_3(x_3^H)}\int_{F_4(x_4^L)}^{F_4(x_4^H)}\int_{F_5(x_5^L)}^{F_5(x_5^H)}\int_{F_6(x_6^L)}^{F_6(x_6^H)}\left(\frac{d^5}{dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)\,dF_5(x_5)dF_6(x_6)}C\left(F_2(x_2),F_3(x_3),F_4(x_4),F_5(x_5),F_6(x_6)\right)\right)$$

$$dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)\,dF_5(x_5)dF_6(x_6)$$

$$= A$$

$$\times\int_{F_2(x_2^L)}^{F_2(x_2^H)}\int_{F_3(x_3^L)}^{F_3(x_3^H)}\int_{F_4(x_4^L)}^{F_4(x_4^H)}\int_{F_5(x_5^L)}^{F_5(x_5^H)}\left(\frac{d^4}{dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)\,dF_5(x_5)}C\left(F_2(x_2),F_3(x_3),F_4(x_4),F_5(x_5),F_6(x_6^H)\right)\right.$$

$$\left.-\frac{d^4}{dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)\,dF_5(x_5)}C\left(F_2(x_2),F_3(x_3),F_4(x_4),F_5(x_5),F_6(x_6^L)\right)\right)dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)\,dF_5(x_5)$$

$$= A\times\int_{F_2(x_2^L)}^{F_2(x_2^H)}\int_{F_3(x_3^L)}^{F_3(x_3^H)}\int_{F_4(x_4^L)}^{F_4(x_4^H)}\left(\left(\frac{d^3}{dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)}C\left(F_2(x_2),F_3(x_3),F_4(x_4)F_5(x_5^H)F_6(x_6^H)\right)\right.\right.$$

$$\left.-\frac{d^3}{dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)}C\left(F_2(x_2)F_3(x_3)F_4(x_4)F_5(x_5^L)F_6(x_6^H)\right)\right)$$

$$+\left(-\frac{d^3}{dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)}C\left(F_2(x_2)F_3(x_3)F_4(x_4)F_5(x_5^H)F_6(x_6^L)\right)\right.$$

$$\left.\left.+\frac{d^3}{dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)}C\left(F_2(x_2)F_3(x_3)F_4(x_4)F_5(x_5^L)F_6(x_6^L)\right)\right)\right)dF_2(x_2)\,dF_3(x_3)\,dF_4(x_4)$$

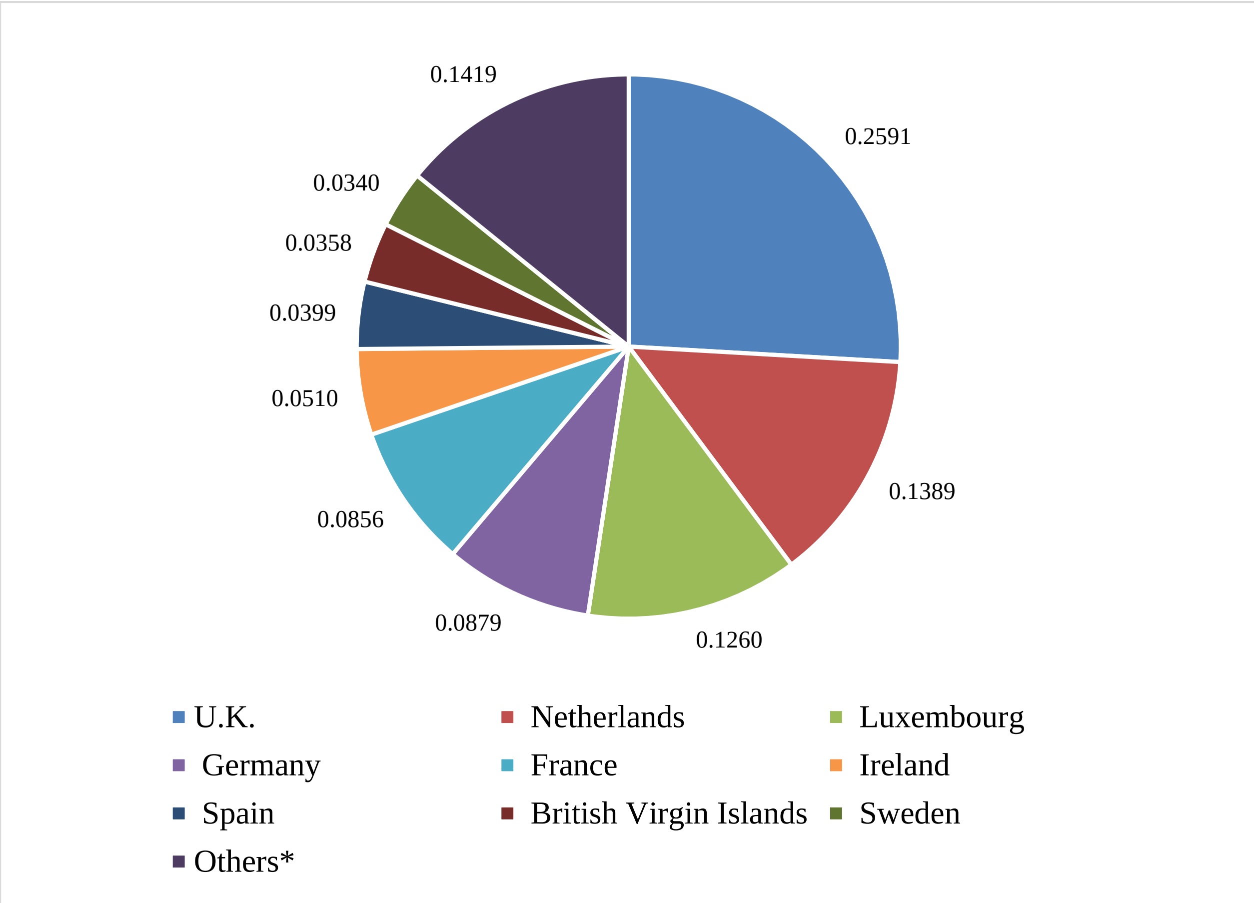
$$
\begin{aligned}
= A \times \int_{F_2(x_2^L)}^{F_2(x_2^H)} \int_{F_3(x_3^L)}^{F_3(x_3^H)} \Bigg(& \frac{d}{dF_2(x_2)dF_3(x_3)} C\left(F_2(x_2)F_3(x_3)F_3(x_4^H)F_5(x_5^H)F_6(x_6^H)\right) \\
& - \frac{d}{dF_2(x_2)dF_3(x_3)} C\left(F_2(x_2)F_3(x_3)F_4(x_4^L)F_5(x_5^H)F_6(x_6^H)\right) \\
& - \frac{d}{dF_2(x_2)dF_3(x_3)} C\left(F_2(x_2)F_3(x_3)F_4(x_4^H)F_5(x_5^L)F_6(x_6^H)\right) \\
& + \frac{d}{dF_2(x_2)dF_3(x_3)} C\left(F_2(x_2)F_3(x_3)F_4(x_4^L)F_5(x_5^L)F_6(x_6^H)\right) \\
& - \frac{d}{dF_2(x_2)dF_3(x_3)} C\left(F_2(x_2)F_3(x_3)F_4(x_4^H)F_5(x_5^H)F_6(x_6^L)\right) \\
& + \frac{d}{dF_2(x_2)\, dF_3(x_3)} C\left(F_2(x_2)F_3(x_3)F_4(x_4^L)F_5(x_5^H)F_6(x_6^L)\right) \\
& + \frac{d}{dF_2(x_2)dF_3(x_3)} C\left(F_2(x_2)F_3(x_3)F_4(x_4^H)F_5(x_5^L)F_6(x_6^L)\right) \\
& - \frac{d}{dF_2(x_2)\, dF_3(x_3)} C\left(F_2(x_2)F_3(x_3)F_4(x_4^L)F_5(x_5^L)F_6(x_6^L)\right)\Bigg) dF_2(x_2)dF_3(x_3)
\end{aligned}
$$

$$
\begin{aligned}
= A \times \int_{F_2(x_2^L)}^{F_2(x_2^H)} \Bigg(& \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^H)F_4(x_4^H)F_5(x_5^H)F_6(x_6^H)\right) - \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^L)F_4(x_4^H)F_5(x_5^H)F_6(x_6^H)\right) \\
& - \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^H)F_4(x_4^L)F_5(x_5^H)F_6(x_6^H)\right) + \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^L)F_4(x_4^L)F_5(x_5^H)F_6(x_6^H)\right) \\
& - \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^H)F_4(x_4^H)F_5(x_5^L)F_6(x_6^H)\right) + \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^L)F_4(x_4^H)F_5(x_5^L)F_6(x_6^H)\right) \\
& + \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^H)F_4(x_4^L)F_5(x_5^L)F_6(x_6^H)\right) - \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^L)F_4(x_4^L)F_5(x_5^L)F_6(x_6^H)\right) \\
& - \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^H)F_4(x_4^H)F_5(x_5^H)F_6(x_6^L)\right) + \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^L)F_4(x_4^H)F_5(x_5^H)F_6(x_6^L)\right) \\
& + \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^H)F_4(x_4^L)F_5(x_5^H)F_6(x_6^L)\right) - \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^L)F_4(x_4^L)F_5(x_5^H)F_6(x_6^L)\right) \\
& + \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^H)F_4(x_4^H)F_5(x_5^L)F_6(x_6^L)\right) - \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^L)F_4(x_4^H)F_5(x_5^L)F_6(x_6^L)\right) \\
& - \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^H)F_4(x_4^L)F_5(x_5^L)F_6(x_6^L)\right) \\
& + \frac{d}{dF_2(x_2)} C\left(F_2(x_2)F_3(x_3^L)F_4(x_4^L)F_5(x_5^L)F_6(x_6^L)\right)\Bigg) dF_2(x_2)
\end{aligned}
$$

$= \cdots = Equation\ (A.1).$ ■

Appendix 8

The S&P report (2016) provides the following regional definition for 'Europe': "Austria, Belgium, British Virgin Islands, Bulgaria, Channel Islands, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Gibraltar, Greece, Guernsey, Hungary, Iceland, Ireland, Isle of Man, Italy, Jersey, Latvia, Liechtenstein, Lithuania, Luxembourg, Malta, Monaco, Montenegro, Netherlands, Norway Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, and the U.K". Some of these countries accommodate significantly more S&P rated companies than others. More specifically, from Figure A.5, which presents the distribution of European corporates rated by S&P across countries, it can be concluded that the vast majority of the rated companies are situated in Western European countries.



* The rest category 'Others' contains the following countries: Switzerland, Italy, Belgium, Austria, Denmark, Finland, Norway, Jersey, Greece, Portugal, Czech Republic, Poland, Guernsey, Cyprus, Romania, Bulgaria, Croatia, Hungary, Malta, Slovenia, Estonia, Iceland, Lithuania, Slovakia.

(Data retrieved from: https://www.standardandpoors.com/en_US/web/guest/entity-browse)

Figure A.5. Distribution of European corporates rated by S&P across countries.

Appendix 9

Table A.2. S&P's average one-year European migration matrix over 1981-2016 (Retrieved from: S&P, 2016).

	AAA	AA	A	BBB	BB	B	CCC-C	D
AAA	0.87145	0.11760	0.00650	0.00220	0.00000	0.00000	0.00220	0.00000
AA	0.00300	0.88270	0.10830	0.00600	0.00000	0.00000	0.00000	0.00000
A	0.00010	0.02020	0.91310	0.06400	0.00200	0.00011	0.00000	0.00042
BBB	0.00000	0.00108	0.04605	0.90490	0.04200	0.00400	0.00108	0.00086
BB	0.00000	0.00000	0.00104	0.06030	0.84090	0.08827	0.00474	0.00474
B	0.00000	0.00000	0.00047	0.00434	0.07807	0.83610	0.05130	0.02970
CCC-C	0.00000	0.00000	0.00000	0.00000	0.00000	0.16757	0.50787	0.32456

Table A.2 presents the average one-year migration probability of European corporations over 1981-2016. This matrix is transformed into a matrix presenting the cumulative migration probabilities for each credit rating.

Table A.3. Cumulative migration probability for individual credit ratings.

	D	CCC-C	B	BB	BBB	A	AA	AAA
AAA	0	0.00220	0.00220	0.00220	0.00440	0.01090	0.12850	1
AA	0	0	0	0	0.00600	0.11430	0.99700	1
A	0.00042	0.00042	0.00053	0.00253	0.06653	0.97963	0.99983	1
BBB	0.00086	0.00194	0.00594	0.04794	0.95284	0.99889	1	1
BB	0.00474	0.00947	0.09775	0.93865	0.99895	1	1	1
B	0.02970	0.08100	0.91710	0.99517	0.99951	1	1	1
CCC/C	0.32456	0.83243	1	1	1	1	1	1

In order to construct the investment pool and speculative pool of obligors, the contribution of each credit rating within the larger pools is a necessary input. The distribution of companies in each of the two credit rating pools are provided in Table A.4.

Table A.4. Fraction of each credit rating in the two pools (Adapted from: S&P, 2014b).

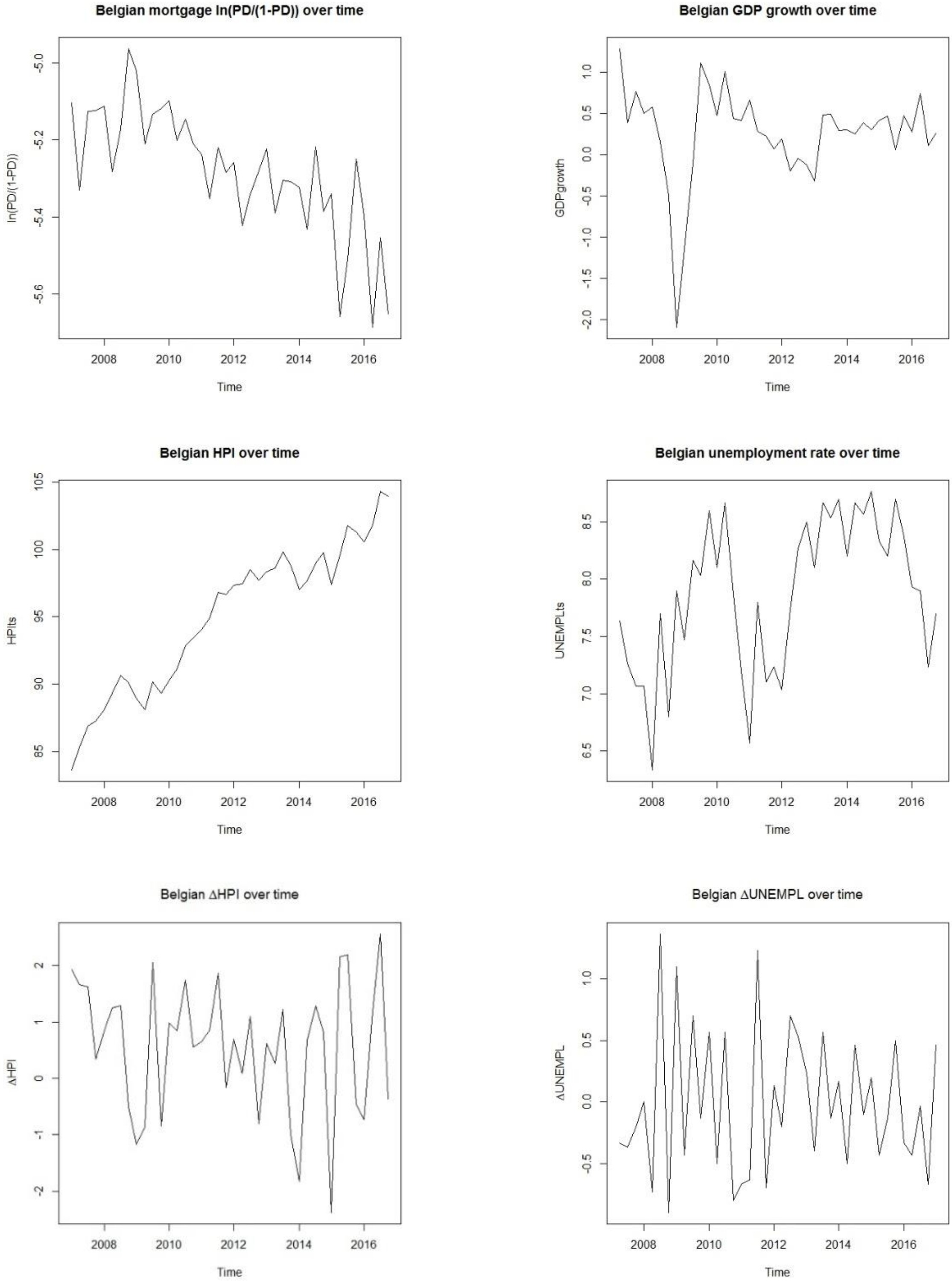
	Investment grade				Speculative grade		
Credit rating	AAA	AA	A	BBB	BB	B	CCC-C
Percentage	0.038411	0.166887	0.353642	0.441060	0.313492	0.635913	0.050595

Given the cumulative migration matrix of the individual ratings (Table A.3) and the composition of the two pools (Table A.4), the cumulative migration matrix of the pools can be computed by weighing the numbers of Table A.3 by the numbers of Table A.4. The result is presented in Table A.5.

Table A.5. Cumulative migration probability of the two pools.

	D	CCC-C	B	BB	BBB	A	AA	AAA
Investment	0.00053	0.00109	0.00289	0.02212	0.44496	0.80650	0.96595	1
Speculative	0.03679	0.09660	0.66443	0.97769	0.99936	1	1	1

Appendix 10



The plots above are helpful to decide whether or not the ADF test equation should include a trend term. Only for $\ln \frac{PDmortgages}{1-PD_{mortgages}}$ and HPI a trend term is included. $GDPgrowth$ looks stationary. First differences are computed to obtain stationarity for the time series HPI and $UNEMPL$.

Figure A.6. (Stationary transformations of) variables included in the dynamic lag model.

Table A.6. ADF tests macro-economic risk factors.

	ADF test stat.	p-value
$\ln\left(\frac{PD}{1-PD}\right)$	-6.2435	5.974×10^{-5}
$GDPgrowth$	-2.9657	0.04876
$PC1$	-4.6136	0.0007972
$PC2$	-3.8085	0.006661
HPI	-2.5441	0.3065
$UNEMPL$	-2.2887	0.1812
ΔHPI	-3.1695	0.03106
$\Delta UNEMPL$	-9.3838	1.285×10^{-8}

Appendix 11

Table A.7. Best dynamic lags model according to BIC/AIC/R^2/R^2_{adj}.

	Model id.	Number of lags of each regressor ϵ	$GDPgrowth$	ΔHPI	$\Delta UNEMP$	BIC	AIC	R^2	R^2_{adj}
Best models according to BIC	2	/	/	/	t-2	-64.68	-73.12	0.3292	0.2700
	7	/	/	t	t-2	-64.31	-74.45	0.3854	0.3109
	13	/	/	t-1	t-2	-63.12	-74.94	0.4245	0.3346
	36	/	t	/	t-2	-62.77	-72.90	0.3599	0.2823
	5	t	/	t	t	-62.76	-69.51	0.2258	0.1839
Best models according to AIC	58	/	t	t-3	t	-61.84	-75.35	0.5052	0.4062
	13	/	/	t-1	t-2	-63.12	-74.94	0.4245	0.3346
	7	/	/	t	t-2	-64.31	-74.45	0.3854	0.3109
	48	/	t	t-1	t-2	-60.72	-74.23	0.4437	0.3361
	59	/	t	t-3	t-1	-58.75	-73.95	0.5132	0.3957
Best models according to R^2	1039	t-4	t-3	t-4	t-4	-20.44	-54.21	0.5841	0.1437
	1074	t-4	t-4	t-4	t-3	-20.43	-54.21	0.5840	0.1436
	1038	t-4	t-3	t-4	t-3	-24.12	-56.21	0.5840	0.1911
	859	t-3	t-4	t-4	t-4	-20.42	-54.20	0.5839	0.1434
	823	t-3	t-3	t-4	t-4	-24.10	-56.19	0.5838	0.1907
Best models according to R^2_{adj}	64	/	t	t-4	t	-56.64	-71.84	0.5303	0.4129
	65	/	t	t-4	t-1	-53.95	-70.84	0.5432	0.4078
	58	/	t	t-3	t	-61.84	-75.35	0.5052	0.4062
	61	/	t	t-3	t-3	-54.09	-72.66	0.5476	0.3968
	59	/	t	t-3	t-1	-58.75	-73.95	0.5132	0.3957

Appendix 12

Table A.8 provides the results of the Augmented Dickey-Fuller tests for the risk-free interest rates, the percentage changes in the interest rates (computed by Equation (20)) and the regular changes in the interest rates (computed by Equation (21)) over a ranging maturity.

Table A.8. ADF tests on $r_m(t), \%\Delta r_m(t)$ and $\Delta r_m(t)$.

Maturity	3M	6M	1YR	2YR	3YR	4YR	5YR	6YR	7YR	8YR	9YR	10YR	15YR	30YR
Augmented Dickey-Fuller test for interest rates, $r(t)$.														
ADF test stat.	-6.7562	-6.5664	-8.3052	-6.0681	-5.2447	-4.9478	-2.7455	-2.9868	-2.4495	-2.3344	-2.3275	-2.3567	-2.5993	-2.7207
p-value	<0.0001	<0.0001	<0.0001	<0.0001	0.0008	0.0017	0.2261	0.1504	0.3494	0.4053	0.4088	0.3942	0.2829	0.2351
Augmented Dickey-Fuller test for percentage changes in interest rates, $\%\Delta r(t)$.														
ADF test stat.	-5.1179	-5.8036	-4.8609	-5.0029	-5.1518	-4.0007	-5.8081	-6.6509	-2.6949	-4.8889	-0.9032	-3.3626	-6.2283	-6.8519
p-value	0.0002	<0.0001	0.0004	0.0003	0.0002	0.0041	<0.0001	<0.0001	0.08562	0.0004	0.7746	0.0199	<0.0001	<0.0001
Augmented Dickey-Fuller test for regular changes in interest rates, $\Delta r(t)$.														
ADF test stat.	-4.2709	-4.4718	-4.2796	-4.4813	-4.576	-4.3709	-4.9641	-4.6249	-5.3203	-4.9941	-5.0749	-5.3437	-5.8436	-6.4876
p-value	0.0020	0.0012	0.0020	0.0011	0.0009	0.0015	0.0003	0.0008	0.0001	0.0003	0.0002	0.0001	<0.0001	<0.0001

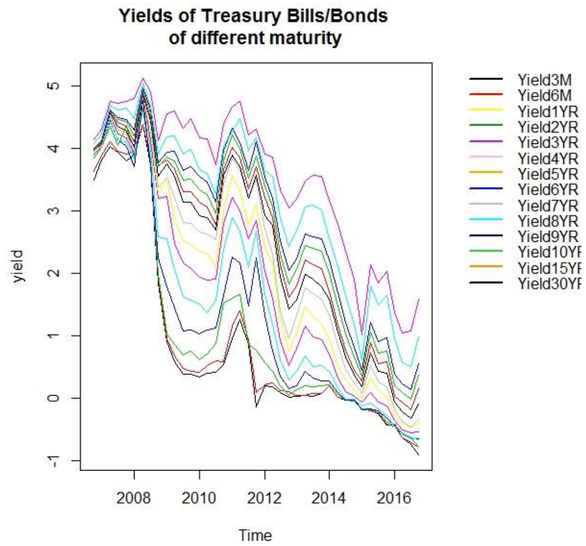


Figure A.7. Interest rates of Belgian benchmark bonds with different maturities.

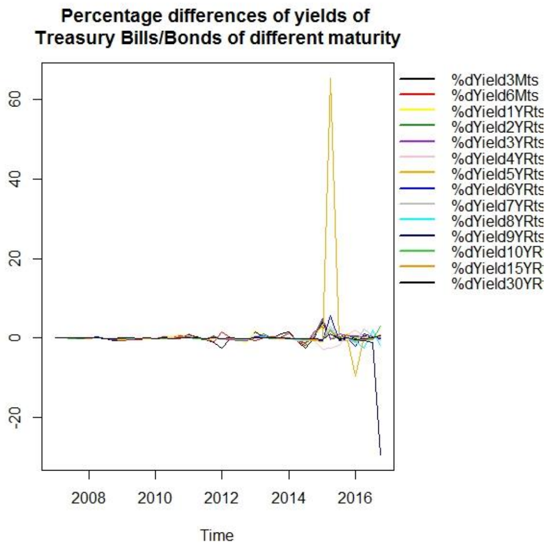


Figure A.8. Percentage changes in interest rates of Belgian benchmark bonds with different maturities.

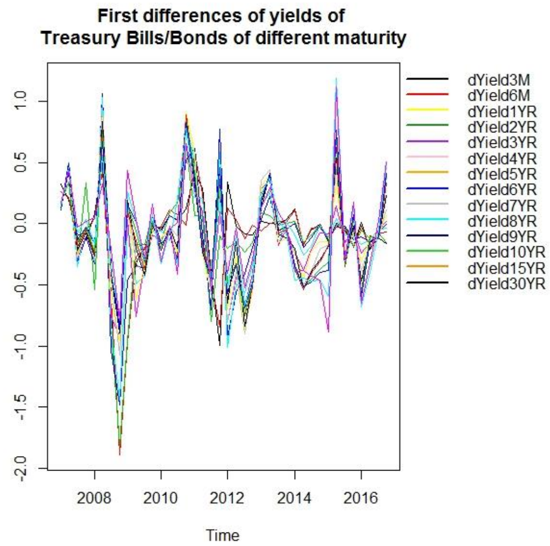


Figure A.9. First difference in interest rates of Belgian benchmark bonds with different maturities.

Appendix 13

Table A.9. Parameters t-copula (df=4) for $GDPgrowth, PC_1, PC_2$.

	$GDPgrowth$	PC_1	PC_2
$GDPgrowth$	1	- 0.35442	- 0.08506
PC_1		1	- 0.42399
PC_2			1

Table A.10. Parameters t-copula (df=5) for all factors.

	$GDPgrowth$	PC_1	PC_2	ΔHPI	$\Delta UNEMPL$
$GDPgrowth$	1	- 0.34144	- 0.05091	0.20730	- 0.11779
PC_1		1	- 0.45367	-0.02701	0.55659
PC_2			1	- 0.01021	- 0.23173
ΔHPI				1	0.15266
$\Delta UNEMPL$					1

Appendix 14

Table A.11. Number of RSTSs determined.

	Investment grade		Speculative grade	
	Normal	Normal & GPD	Normal	Normal & GPD
Number of RSTSs	14512	8192	75224	91448

Appendix 15

Table A.12. Most different RSTSs for the four settings.

Initial rating	Marg. distr.	Z	$GDPgrowth$ (%)	PC_1 (%)	PC_2 (%)	ΔHPI (%)	$\Delta UNEMPL$ (%)
Speculative grade	Normal	0.5	-0.0332	-4.8614	0.7439	-0.0275	-0.3052
		0.5	0.5441	-4.8614	-0.7439	-0.0275	-0.3052
		-0.5	0.5441	-4.8614	-0.7439	1.1400	-0.3052
	GPD	0.5	-0.0332	-4.8614	-0.7439	-0.0275	-0.3052
		-0.5	-0.0332	-4.8614	-0.7439	-1.1949	-0.8807
		-0.5	-0.6105	-4.8614	0.7439	-0.0275	-0.8807
Investment grade	Normal	-0.5	0.5441	11.3433	-5.2072	4.6422	0.2702
		-0.5	-1.1877	11.3433	-5.2072	3.4748	1.4212
		0.5	-0.0332	11.3433	-5.2072	-1.1945	1.4212
	GPD	0.5	-1.1877	11.3433	-5.2072	3.4748	0.8457
		-0.5	-0.6105	8.1024	-5.2072	4.6422	1.9967
		0.5	-0.0332	11.3433	-5.2072	-0.0275	1.9967

FACULTY OF BUSINESS AND ECONOMICS
Naamsestraat 69 bus 3500
3000 LEUVEN, België
tel. + 32 16 32 66 12
fax + 32 16 32 67 91
info@econ.kuleuven.be
www.econ.kuleuven.be