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FACULTY OF ECONOMICS AND BUSINESS

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# Modelling macroeconomic stress test and its impact on credit portfolio quality

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Thesis submitted to obtain
the degree of

MASTER IN DE TOEGEPASTE ECONOMISCHE WETENSCHAPPEN:
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# Modelling macroeconomic stress test and its impact on credit portfolio quality

This paper describes a multi-factor, Markov chain model as proposed by the BOJ and implements it in a framework for macro stress testing credit risk of a corporate portfolio mainly focusing on the Eurozone as a whole. The framework starts with the creation of transition matrices based on the duration approach and ends with a simulation of credit migrations to assess credit portfolio quality. This assessment takes into account recent capital requirements of the Basel accords and computes the RWAs using the Standardized Approach. The framework proves to be very flexible as it is able to capture the main credit migrations observed in recent history and can accommodate a variety of stress testing scenarios. Furthermore the model is able to predict reasonable outcomes. The approach is carefully outlined and deals with all the main issues of a stress testing exercise. The paper concludes with the execution of the framework and a discussion of the main results.

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Finally, I would like to thank my mother, my father and my brother for supporting me all my life and as it is the case for everything I do, this thesis I dedicate to them.

## Abbreviations

| | |
|---|---|
| **(A)-IRB** | (Advanced) Internal Ratings-Based approach |
| **BOJ** | Bank of Japan |
| **CET1** | Common Equity Tier 1 |
| **CPI** | Consumer Price Index |
| **DE** | Differential Evolution |
| **EAD** | Exposure At Default |
| **EBA** | European Banking Authority |
| **EU** | European Union |
| **GDP** | Gross Domestic Product |
| **HICP** | Harmonized Index of Consumer Prices |
| **LGD** | Loss Given Default |
| **PD** | Probability of Default |
| **PIT** | Point-In-Time |
| **RMSE** | Root Mean Squared Error |
| **RWA** | Risk Weighted Assets |
| **SME** | Small and Medium-sized Enterprises |
| **TTC** | Through-The-Cycle |
| **VAR** | Vector Autoregression |

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# Introduction

In response to the financial crisis in 2007-2009, assessing a bank's financial health in various negative economic scenarios, a practice better known as stress testing, has become increasingly important. This trend for developing models that are able to relate adverse economic scenarios and their impact on a bank's performance is driven by regulatory authorities who seek to strengthen risk management and capital adequacy of banks in order to avoid a new financial crisis.

In the European Union, the EBA plays a key role in the implementation of the progressively demanding Basel frameworks[1] and the exercise of the EU wide stress tests[2] which are out to be held periodically. As a result, banks are forced to integrate stress testing as an important part of their business model and strategic planning.
The implementation of the Basel II capital adequacy rules[3] granted banks some credit risk measurement techniques such as the Standardized and the A-IRB approach. Both methods are ratings-based and aim to quantify capital requirements for credit risk. The main difference between these methods is that the Standardized Approach depends heavily on external credit ratings provided by credit rating agencies whereas under the A-IRB approach banks are allowed to use their own quantitative methods to determine the creditworthiness of their counterparties. Recent proposals of the Basel committee within the Basel IV suggest a further increase in capital requirements, simplicity and comparability across financial institutions resulting in a distancing of the A-IRB approach. Although these techniques increase comparability, no clear way to relate these measurements to macroeconomic conditions are suggested. In addition, the developed scenarios remain arbitrary.

The literature on credit risk modelling has grown along two main strands. A first strand has dealt with the issue of modelling and predicting financial distress and rating migrations. Examples include Altman (1968), Ohlson (1980), Fons (1994), Wilson (1997a, 1997b), Belkin et al. (1998), and Duffie (1998). A second strand has attempted to build pricing models able to value credit derivatives such as bonds and credit default swaps. Examples include Merton (1974), Vasicek (1984), Jarrow and Turnbull (1995), Duffie and Singleton (1999), Jarrow, Lando and Turnbull (1997), Hull and White (2000, 2001) and Andersson (2007, 2010). Although these strands differ in their application, both are very closely connected and depend on three main variables: the probability of default (PD), the loss given default (LGD) and the exposure given default (EAD). Following Altman (2003), these credit pricing models can be categorized in two main groups.

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[1] The Basel frameworks are constructed by the Basel committee. This is a standard setter for the regulation of banks. The proposed frameworks so far are stated in Basel I, Basel II , Basel III and Basel III.5

[2] The EBA is responsible for the implementation of the Basel frameworks in the EU. In addition it is set to organize periodical EU-wide stress tests in order to assess the resilience of financial institutions. The most recent exercise was held in 2016 and covers over 70% of the national banking industry in the Euro Area

[3] In the Basel II framework, the common tier 1 capital ratio was set at 7% of the total risk weighted assets

A first distinction is what is known in the literature as the Structural Form models which are based on the framework developed by Merton (1974). In his model, still widely known as the 'Merton Distance to Default Model', Merton proposes to calculate the PDs and credit spreads using the value of the firm's assets and default occurs when the market value of a firm's asset is lower than its liabilities. Although using many simplifying assumptions[4] , Merton sets the groundwork for the next generations of structural models proposed by authors as Black and Cox (1976), Geske (1977), Vasicek (1984) and Hull and White (1995) who all have sought to find and model more realistic assumptions. A main drawback of all these structural models is that they assume a two-state economy where companies either survive or default and thus fail to incorporate credit migrations and relative performance. In addition default is supposed to occur with certainty and no unpredicted defaults can occur.

As a response, a second group has emerged known as the Reduced Form models which are pioneered by Fons (1994). In contrast to the Structural Form models, these models do not anchor on the firm's value but depend for the computation of the PDs on (external) ratings and historical default rates of distinct rating classes. Extending his work Jarrow, Lando and Turnbull (1997) introduce the time homogeneous[5] Markovian Model which can capture credit migrations countering the Merton framework in which credit migrations do not affect the price of loans and bonds. The work of Jarrow, Lando and Turnbell was a huge step forward in modelling credit risk and was the beginning of what would grow to a relative large body of literature. Modelling the credit migrations as a time homogeneous Markov Model may be convenient to compute, however authors as Altman (1998), Nickell et al. (2000), Lando and Skødeberg (2002) have shown non-Markovian characteristics such as ratings drift and variations related to the business cycle resulting in more stable higher-rated bonds and an increase in default rates during economic downturn. These findings have initiated a flood of proposed methods in order to find ways to generalize the time homogeneous Markovian Model. Examples include Christensen and al. (2004) who consider the possibility of hidden states for certain rating dynamics in order to determine an underlying ratings drift, similar Giampieri and al. (2005) develop a hidden Markov model to determine the state of the economy, Wei (2003) proposes a Multi-Factor Markov chain model which allows credit rating specific migrations to depend on business cycles and is an extension of Belkin and al.'s (1998) One-Factor model.

In a more recent paper Kuznetsov (2006) combines previous insights with a more affine[6] approach. Kuznetsov estimates a generator matrix which represents the credit migration drift and introduces an independent stochastic time change parameter which controls for the severity of that drift[7]. One drawback of this model is that once the generator drift is estimated, the direction of the rating drift is fixed. The drift itself can either slow down or accelerate depending on the economic cycle, however the direction remains unchanged. This drawback is addressed by Andersson (2010) who makes use of two generator matrices in order to be able to capture both upwards as downwards drift. In addition Andersson (2010) introduces two concepts: rating speed and rating direction. The underlying assumption is that the speed of migrations can differ among different rating classes. The model proposed by Kuznetsov (and by extension Andersson) on the one hand succeeds in capturing the complexity of the credit reality, but on the other hand is computationally challenging.

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[4] For example a company is assumed only to default at maturity and the PDs are known with certainty.

[5] Time homogeneous and thus do not depend on rating history or business cycle.

[6] An affine transformation preserves proportions and collinearity and is a way of mapping transition matrices to a generator matrix.

[7] You can interpret a drift as 'being pushed into a direction'. For transition matrices a drift could be for example: observations of strong movements to CCC rating class.

All the models discussed above lack a clear methodology to relate observed credit migrations (or default rates in the structural models) to macroeconomic conditions and the body of literature dealing with macroeconomic stress tests for credit risk is still sparse. The vast majority is based on Wilson's (1997a, 1997b) credit portfolio model. Wilson introduces the concept of credit migrations and their relation to the health of the economy by means of a shift matrix representing the sensitivity to systematic risk. A major drawback however, is that he fails in specifying an exact method to determine this matrix[8]. Belkin et al. (1998) pick up this idea of an underlying shift variable in their One-Factor Model and outline a clear computational methodology. Based on this shift, Boss (2002) and Virolainen (2004) directly relate the dynamics in the default rate of an economic sector to macroeconomic factors such as GDP, interest rates and measures of corporate indebtedness.

This paper attempts to exercise a macroeconomic stress test solely focusing on corporate exposures in the EU market and adopts the methodology outlined by the BOJ (2009), which is an extension of the Multi-Factor Markov Model proposed by Wei (2003), and attempts to find a clear relation of credit rating dynamics to particular macroeconomic variables. It differs from previous work in several ways. First, it is a stress testing exercise from start to finish and does not depend on external computations such as the construction of transition matrices. Second, it can serve as an overview of the numerous challenges, difficulties and complexities one has to face in performing this kind of exercises and finally in respect to the adopted methodologies, several improvements are proposed ranging from handling missing data to distribution fitting.

The remainder of the paper is structured in two main parts. Part 1 consists of three chapters: Chapter 1 denotes an introduction in Through-The-Cycle and Point-In-Time methodologies followed by the approach used to create transition matrices. In Chapter 2 the framework for macro stress testing is set out followed by an explanation of the Multi-Factor model and its link to macroeconomic variables and concludes with an overview of the capital requirement measures proposed by the Basel committees. The section concludes by a clarification of the approach on the subject of stress testing and portfolio simulation. Part 2 consists of two chapters: Chapter 3 describes the data used in this paper and the associated data problem handling. Chapter 4 discusses the obtained result and the last section concludes.

[8] He did however specify what form this matrix should be.

# Part 1: Methodology

# 1 Building a rating system

This chapter will provide insight on the complexities of transition matrices and their application to credit management and starts with an illustration of the difference between a Through-The-Cycle and Point-In-Time approach followed by a theoretical clarification and the importance of the differential evolution algorithm. Finally a short discussion of the main credit scoring models used in this paper are reviewed.

## 1.1 TTC vs PIT

The IRB approach requires the estimation of an individual estimated PD for each obligor in the portfolio. These PDs are constructed using credit scoring models which will be discussed in the last section of this chapter. Next the PDs are used as inputs for constructing risk buckets. The potential bucketing approaches will be discussed into more details under section 1.2. Once the risk buckets are constructed, obligors that share the same credit quality must be assigned to the same risk bucket and share thus a similar PD or pooled PD which represent the risk of obligors within the group. One has basically the choice between two approaches: the first one is a PIT rating system which evaluates the credit worthiness of a borrower using all current information. Typical for these rating system is that their rating dynamics will be volatile as they are affected by business cycles. In addition the pooled PDs of every rating group usually remains fixed and obligors migrate from one credit group to the next according to changes in their individual PD. The second approach is a TTC rating system which is much more stable and is presumed to be 'business cycle neutral'. In contrast with a PIT rating system the pooled PDs are not presumed to be fixed and can change from period to period. An obligor will then migrate from one rating group to another if their individual PD changes significantly relative to the change in the pooled PDs. Figure 1 and Figure 2 illustrate the TTC and PIT rating systems respectively.

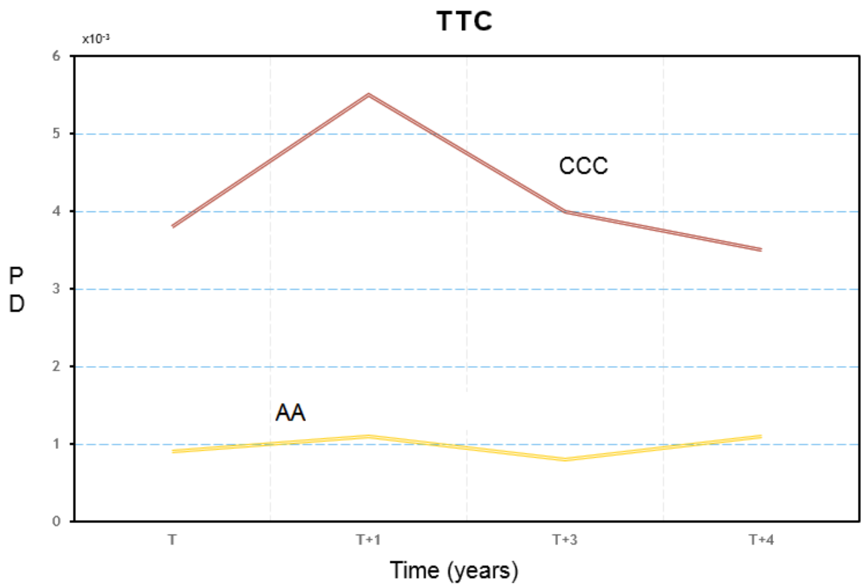


**Figure 1: Illustration of the TTC rating system (Vedran Cengic 2016)**

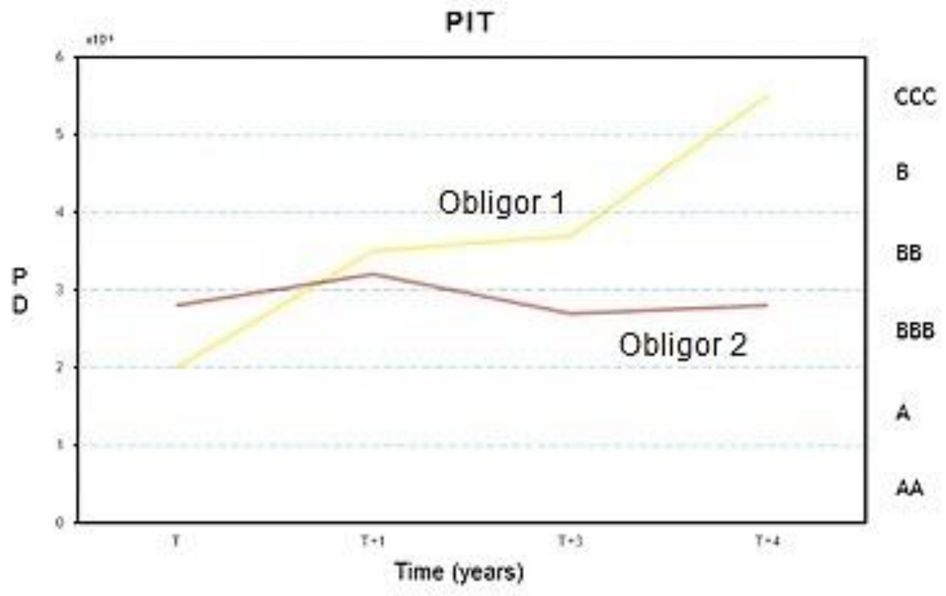


**Figure 2: Illustration of the PIT rating system (Vedran Cengic 2016)**

In Figure 1 the vertical and horizontal axis depict the PD and timeline respectively and two rating groups are illustrated. The yellow line represents the pooled PD of the AA rating bucket and the red line represents the pooled PD the CCC rated obligors. It is clear that both lines move up and down through time and obligors who have a PD close to 0.0009 in year T would be assigned a AA rating while in year T+1 a PD close to 0.0011 would earn a AA rating. A second example can help to illustrate the TTC approach: if an obligor in year T has a PD close to 0.0038 it will be assigned a CCC rating. Similarly, if the year after its PD would increase to 0.0055 it will still earn the CCC rating and in the unlikely case that its PD drops close to 0.0011 it will be assigned a AA rating.

Figure 2 uses the same axes only this time the rating groups are fixed along the vertical axis and the groups are separated from each other by the blue dotted lines. In contrast to Figure 1, the yellow and the red line represent the path of the obligors through time. If for example obligor 1 would have an estimated PD in year 1 between 0.002 and 0.003 it would earn a BBB rating. If its PD would increase the next year to a value between 0.003 and 0.004 a BB rating would be assigned to obligor 1.[9]

Section 2.3 of this paper discusses the capital requirements under the Basel frameworks. In this section it will be clear that choosing the right rating philosophy is vital as under the Standardized Approach, exposures are weighted relative to their associated credit rating in order to compute the RWAs. It is thus crucial to capture the right credit migrations for assigning the right risk weights to the associated exposures. Similar, under the (A)-IRB Approach, the pooled PDs are a very important input in the formula to compute the RWAs ( see Appendix 1).

[9] Please note that these figures serve as mere illustrations and are not intended to represent realistic PD values

## 1.2 Transition matrix and DE algorithm

Before heading into detail on how the risk buckets are computed and why it is an essential part of every rating system, a short introduction on transition matrices follows.

### 1.2.1 Transition matrix

$$P_t = \begin{pmatrix} P_{1,1} & P_{1,3} & \cdots & P_{1,K} \\ P_{2,1} & P_{2,3} & \cdots & P_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ P_{K-1,1} & P_{K-1,2} & \cdots & P_{K\_1,K} \\ 0 & 0 & \cdots & 1 \end{pmatrix} \quad (1)$$

The transition matrix or migration matrix states the probabilities of all possible transitions during a particular period[10]. The rating scale is replaced by a numeric equivalent: {AAA, AA, A, BBB, BB, B, CCC, CC, C, D} <-> {1, 2, 3, 4, 5, 6, 7, K}. $P_{ij,t}$ denotes the probability of a firm in state i in year t migrating to a state j in year $t_{+1}$. The rows represent the current rating whilst the columns represent the future rating. The last row represent the ‘Default’ category which is an absorbing state meaning that no other migrations can occur out of a defaulted state. The elements below the diagonal are probabilities associated with upgrades and the elements above the diagonal the probabilities associated with downgrades. In a TTC approach the elements on the diagonal are expected to have the highest probabilities meaning that the probability of preserving a credit rating in $t_{+1}$ is the highest. In general two main methods for estimating the transition probabilities exist: the cohort estimation and the duration estimation[11]. In the cohort approach the transition probability $P_{ij,t}$ from rating i at $t_{-1}$ to rating j at t is simply the number of transitions $N_{ij,t}$ observed from rating i at $t_{-1}$ to rating j at time t divided by the sum of all transitions $\sum N_{ij,t}$ from rating i at time $t_{-1}$ to all other ratings j at time t. The duration approach counts all rating changes during the observation period and weighs by the time spent in the rating at t in order to obtain the migration intensity. This is then transformed into transition probabilities. Schurmann’s (2004, 2006) suggests a high inefficiency in the cohort approach to capture the real transition probabilities and proves that the duration method is much more efficient.

[10] Can be 1-year transition matrices, 2-years and so on.

[11] In reality there exist different versions of the duration approach, however the main idea remains the same.

### 1.2.2 DE algorithm

As banks must hold regulatory capital based upon the creditworthiness of their obligors under the Basel frameworks, it's of high importance to rightfully assign a rating to an individual obligor. Constructing the range of the rating groups too wide could significantly increase the necessary regulatory capital as more obligors could be assigned a lower rating. Setting them too narrow could understate the required capital[12]. Although a vital element in the construction of transition matrices, the construction of risk bucketing hasn't received much research attention. Possibly because this is considered to be a clustering problem. It's important to note that loan portfolios can consist of multiple thousands of obligors and the optimization is a NP-hard problem. Investing in a powerful algorithm is thus crucial. Several clustering algorithms can be used to solve this problem. Examples include algorithms based on K-means clustering, neural networking, differential evolution and threshold accepting. Although Lyra (2008) proves the threshold acceptance algorithm to be the best choice, this paper sets out the differential evolution algorithm which has a proven efficiency track record and implementation simplicity, Krink (2007),

#### *1.2.2.1 Objective function*

The objective is to find a pooled PD ($\overline{PD}$) for each rating bucket such that the squared error of replacing the PD of an individual borrower with the pooled PD is minimized.

$$\text{Min} \sum_{b} \sum_{i \in b} (PD_i - \overline{PD_b})^2 \tag{2}$$

with i an individual obligor and risk bucket b. The optimization problem has to consider some constraints which were imposed in the Basel II framework. These constraints are:

1. Pooled PDs cannot have a value less than 3 base points.
2. No rating bucket can include more than 35% of the total bank's exposure
3. The rating system must include at least 7 rating buckets[13]

After finding the optimal pooled PDs, every rating bucket will consist of a number of borrowers for which the total sum of their squared errors is minimal. This allows to determine min $PD_b$ and max $PD_b$ for $b = 1 \dots 7$. Borrowers can then be classified to an appropriate rating group according to their individual PD. For example, the first bucket (that of the AAA ratings) which has the lowest pooled PD is the interval starting from the minimum PD observed in the sample in year t up to the maximum PD in the rating bucket set by solving the optimization problem and thus the first bucket will be $[\min(PD_{sample,t}), \max(PD_{b1,t})]$. Likewise, the second bucket will range in the interval $]\max(PD_{b1,t}), \max(PD_{b2,t})]$. The other buckets are constructed in similar fashion resulting in $]\max(PD_{b6,t}), \max(PD_{sample,t})\ ]$ for the last rating group.

[12] It depends on which rating groups are set wide. Setting the range of the AAA rating group very wide will result in low RWAs and thus low required capital.

[13] The determination of the optimal number of rating buckets is a subject on its own. To increase comparability with the S&P transition matrices a total of 7 ratings (the default state excluded) are assumed throughout this paper.

### 1.2.2.2 How it works

To help visualize the procedure, the objective minimization is done in the real vector space $R^2$. This is a very similar approach only now the Euclidean distance to the centers of a pool ($\overline{PD}$) is minimized instead of the variance. The algorithm starts with an initial basic solution: a number of random vectors representing the centers of each bucket in the space $\left\{\left(\mathrm{x}, \min\left(PD_{sample}\right)\right); \left(\mathrm{x}, \max\left(PD_{sample}\right)\right)\right\}$.

In each iteration, candidate solutions are generated. These are again all vectors in the space $\{(\mathrm{x}, \min(PD_{sample})); (\mathrm{x}, \max(PD_{sample}))\}$ .The algorithm iterates a specified number of times which must be set ad hoc and each time it generates a sample of possible solutions. The size of this sample must be specified ad hoc as well. To combine the generated candidate solutions to new candidate solutions, three possible strategies are tested. These are known as: DE/RAND/1, DE/BEST/1 and DE/RAND-TO-BEST/1. Formally:

1. $\mathrm{DE/RAND/1}: v_i = x_{r1} + F_1(x_{r2} - x_{r3})$ (3)
2. $\mathrm{DE/BEST/1}: v_i = x_{best} + F_1(x_{r2} - x_{r3})$
3. $\mathrm{DE/RAND-TO-BEST/1}: v_i = x_{r1} + F_1(x_{r2} - x_{r3}) + F_2(x_{best} - x_{r1})$

where $x_{ri}$ represents a random candidate solution, $x_{best}$ the best solution found so far and $v_i$ a new candidate solution generated from the random solutions $x_{ri}$. $F_1$ and $F_2$ are called the scaling factors and needs to be set ad hoc. Usually these scaling factors range between [0.2, 0.8]. Finally a parameter CR called the crossover probability has to be specified. For each generation of possible candidate solutions, in each iteration, there exist 2 possible outcomes:

1. One of the strategies discussed above is applied and evaluated in the objective function.
2. No strategies are applied and the candidate is evaluated immediately.

The CR parameter thus specifies the probability of exercising one of the strategies. All of the parameters discussed above generally result from extensive testing and depend on the predefined level of accuracy. Likewise the choice of strategy depends on sample sizes and accuracy preferences. Following R. Storn (1997) choosing the DE/BEST/1 strategy should lead to the best results. As previously specified, the best solution is the solution that minimizes the Euclidean distance to the generated centers. Each time a better solution is found, the current best solution is replaced. As a result, the bucket intervals or thresholds are obtained and ratings are assigned to each borrower belonging to a bucket. For example AAA rated companies are those with the lowest PD. If the PD of a particular corporate which is calculated by a credit scoring model (see 1.3) is between the thresholds of the bucket with the lowest mean or center, obtained by applying the DE-algorithm, it receives an AAA rating. Figure 4 illustrates how the DE algorithm applies the different strategies discussed above in the search space and eventually finds a (sub) optimal solution.

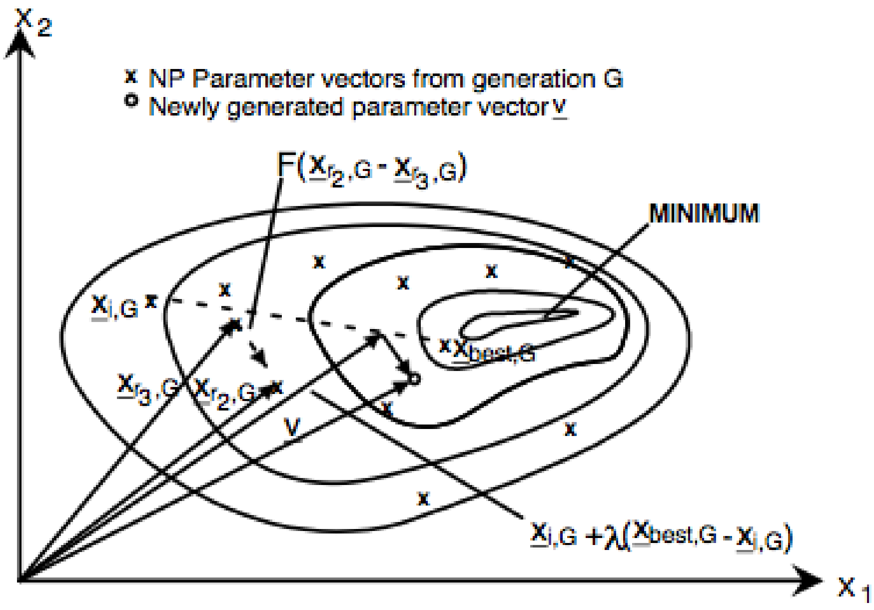


**Figure 3: Search space of the DE algorithm (R. Storn 1997)**

## 1.3 Credit scoring models

In order to assign a particular credit rating to a borrower, one has to have some indication of borrower's credit quality. To date there exist several (publicly available) credit scoring models to help deal with this issue. In addition, most of these credit scoring models have found ways to transform the obtained score to a PD. The vast body of literature has focused on the prediction of financial distress of listed corporates. Very few examples exist of predicting sovereign distress or SMEs. This section will introduce 2 of the main publicly available credit scoring models to date: the Z-Score Altman (1968), and the O-score Ohlson (1980) followed by a more recent model: the CHS-model Campbell, Hilscher, Szilagy (2010). The choice for outlying these models is inspired by their proven track record and comparability: all these models predict financial stress in a logit[14] context, and rely on accounting data; the CHS-model in addition uses market data.

### 1.3.1 Z-Score

The Z-score was created by Altman in 1968. Much is written about this scoring model and although it is the oldest model considered in this paper, it is still the most widely known and used. This paper uses a slightly modified logit version of the original Z-score, to increase comparability with the other scoring models:

$$Z = 7.72 + 0 * \frac{WK}{TA} - 0.52 * \frac{RE}{TA} - 3.61 * \frac{EBIT}{TA} + 1.34 * \frac{ME}{BL} + 0.51 * Size + 0.183\ Age \qquad (4)$$

[14] In the case of the Z-score a more recent version Altman (2004) of the original Z-score is used as the last does not consider the logit transformation.

with WK net working capital, RE retained earnings, TA total assets, EBIT earnings before interest and tax, ME market value of equity, BL book value of liabilities and Age the number of years since a firm was first rated by an agency. Age depends on when all the data of the corporate was first available. Size is total liabilities normalized by total market value of the FTSE100 index. The Z-score is transformed to a PD according to:

$$Z = \log(\frac{P_i}{1 - P_i}) \tag{5}$$

with $P_i$ the probability of a corporate going bankrupt within one year.

### 1.3.2 O-Score

The O-score was developed by Ohlson in 1980 and is an alternative to the Z-score. It has an excellent record in predicting financial distress and is calculated as follows:

$$O = -1.32 - 0.407 * \log(TA_t) + 6.03 * \frac{TL_t}{TA_t} - 1.43 * \frac{WC_t}{TA_t} + 0.0757 * \frac{CL_t}{CA_t} - 1.72 * X - 2.37 * \frac{NI_t}{TA_t} - 1.83 * \frac{FFO_t}{TL_t} + 0.285 * Y - 0.521 * \frac{NI_t - NI_{t-1}}{|NI_t| + |NI_{t-1}|} \tag{6}$$

with TA total assets, TL total liabilities, WC net working capital, CL current liabilities, CA current assets, NI net income and FFO funds for operations. X is a dummy that equals 1 if a corporate has more liabilities than assets; 0 otherwise. Y is a dummy that equals 1 if there was a net loss in the previous 2 years; 0 otherwise. The O-score is transformed to a PD in a similar manner as the Z-score.

### 1.3.3 CHS-Model

The CHS model was created by Campbell, Hilscher, and Szilagy in 2010. In contrast to the previous models, CHS includes data on a quarterly and monthly basis.

$$\begin{aligned} CHS = {} & -8.87 - 20.12 * NIMTAAVG_{t-1,t-12} + 1.6 * \frac{Total\ Liabilities_{it}}{ME_{it} + Total\ Liabilities_{it}} - 2.27 \\ & * \frac{Cash\ and\ short\ term\ investments_{it}}{ME_{it} + Total\ Liabilities_{it}} - 7.88 * EXRETAVG_{t-1,t-12} \\ & + 1.55 * SIGMA_{i,t-1,t-3} \\ & - 0.005 * \log\left(\frac{ME_{it}}{Total\ FTSE\ 100\ Market\ value_t}\right) \\ & + 0.07 * \frac{ME_{it}}{BE_{it} + 0.1 * (ME_{it} - BE_{it})} - 0.09 * Price \end{aligned} \tag{7}$$

with ME the market value of equity, Price the stock price of the company, capped at €13 (prices above €13 are assumed not to have any impact). NIMTAAVG is the weighted average net income in the most recent quarters, EXRETAVG is the weighted average excess stock return relative to the stock return of the FTSE100 index, BE is the book value of equity and SIGMA is the recent volatility of the shares.

# 2 Macro Stress test

This chapter outlines the methodology for macro stress testing following Wei (2003) and the BOJ (2009). First the overall framework is disclosed. Subsequently, the MF model is described in detail. This is an attractive model and has (aside from the ease of implementation) many advantages. First, it is able to model downgrades as well as upgrades whilst a lot of models can only model one of the two Andersson (2010). Second, it provides a clear and understandable way to link the migrations dynamics with macroeconomic variables. Many of the models today use a more mathematical based approach where there is no clear link to the macroeconomic environment. Finally, it is able to measure the sensitivity of each rating class to the macroeconomic variables. This is a very interesting feature as one can assume that different rating classes react to macroeconomic shocks in different ways. Next, the interesting characteristics of the VAR model are reviewed. The chapter concludes with brief discussion of the relevant parts of the Basel frameworks and presents a simulation approach of a credit portfolio.

## 2.1 Framework

First, well-known credit scoring models are applied each year during the observed period to a large sample of corporates across the Eurozone. In order to assign a credit rating and to achieve credit rating stability (see 1.2) it is necessary to relate a particular score to all other scores each year and to find the optimal pooled PD of each rating bucket. This is done using the DE-algorithm.

Second, 1-year transition matrices for borrower classifications are constructed each year during the period as well as a long time average over the same period using the duration approach.

Third, a multifactor model is constructed following Wei's (2003) procedure: each year deviations in every rating category from the long time average transition matrix are estimated and the systematic factor driving these deviations is extracted.

Fourth, these deviations or shifts from the long time average are then regressed against a sample of macroeconomic variables in order to determine the most significant predictors affecting the common factor determined in the previous step.

Fifth, a VAR model is constructed using several macroeconomic variables in order to capture the responses of the predictors determined in the fourth step to shocks in the macroeconomic variables of the VAR model.

Sixth, several stress test scenarios are set up and their implied deviations to the long term transition matrix are estimated using the model obtained in step 4 and their implied transition matrices are constructed.

Finally, the CET1- ratio is simulated of a portfolio of borrowers based on the implied transition matrices obtained in the previous step and the computation of the RWAs using the standardized approach. In this simulation the pooled PDs observed in every year are averaged out to obtain the long time average pooled PD in every rating category.

## 2.2 Modified Multi-Factor Markov Chain Model

### 2.2.1 The S-Matrix

The first step of the model is to establish some sort of mapping in order to translate the long time average transition matrix into a matrix of credit scores. Wei motivates this as mapping future returns to possible ratings with higher ratings associated with higher returns. Wei names this scoring matrix in his paper the Z-matrix, however to avoid confusion with one of the credit scoring models in section 1.3 a different name is used: the S-matrix.

For each rating class in the long term transition matrix, a cumulative distribution is fitted. Whereas Wei uses the normal distribution, this study uses the log logistic distribution for its better fit. However to illustrate the procedure the normal distribution is used as well[15]. Starting from the last column, the mapping to the S-matrix is done by creating bins where the sum of the transition probabilities from the last column up to the column considered is mapped using the inverted cumulative distribution. An example helps to illustrate this: the transition probabilities of BBB to {AAA, AA, A, BBB, BB, B, CCC, CC, C, D} is respectively: [0,0 0,1 4,2 91,3 3,4 0,7 0,1 0,2]. De default probability of 0.2% or 0.002 corresponds to all negative values up to $\Phi^{-1}$ (0.002) which for the normal distribution equals to 2.878. So the first bin is then : $]-\infty,-2.878]$. For the next bin: the probability of a BBB rating migrating to B or CCC, CC, C is 0.1% + 0.3% = 0.4%. Using the same procedure: $\Phi^{-1}$ (0.004) = 2.652 and the next bin is $]-2.878,-2.652]$ and so on. Important to note is that the S-matrix is a (K-1) x (K-1) matrix as there is no need to convert the last row of the transition matrix eq. (1) and because the upper limit of the AA bin is the lower limit of the AAA bin. Doing this for all rating classes results in:

$$S = \begin{pmatrix} S_{1,2} & S_{1,3} & \cdots & S_{1,K} \\ S_{2,2} & S_{2,3} & \cdots & S_{2,K} \\ \vdots & \vdots & \ddots & \vdots \\ S_{K-1,2} & S_{K-1,3} & \cdots & S_{K-1,K} \end{pmatrix} \quad (8)$$

where

$$S_{i,j} = \phi^{-1}(\sum_{k=K}^{j} P_{i,k})$$

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[15] This is a similar approach to J.P Morgan's CreditMetrics™ which uses the normal distribution as well. Although this paper proves the log logistic distribution to be a better fit, the normal distribution is used as an illustration of the procedure due to its numerical simplicity. Using different meaningful distributions will have no impact on the procedure.

### 2.2.2 Shifting the S-Matrix

After obtaining the S-matrix from the long time average the goal is to repeat that same procedure for all the 1-year transition matrices, $S_t$, and to measure the deviations in each year in every rating category from $\bar{S}_{ij}$. In order to obtain the deviation from the average the following equation is determined:

$$Y_{ij,t} = a_i x_t + \sqrt{1 - a_i^2}\, \varepsilon_{ij,t}[16] \tag{9}$$

where $Y_{ij}$ represents the deviation of element $i, j$ in $S_t$ from $\bar{S}_{ij}$ in year t, $\alpha_i$ the sensitivity of the rating category i to the common factor x in year t and $\varepsilon_{ij}$ the non-systematic idiosyncratic factors. Furthermore the correlation between $y_{ij}$ and $y_{kl}$ is assumed to be the same and equals to $\alpha^2$ for every $i, j, k, l$ with $i \neq k$.

The rationale behind eq.(9) is that empirical evidence BOJ (2009), indicates that changes in default rates differ among rating classes with higher changes in the lower rating classes which implies different sensitivities ($\alpha_i$) to changes in systematic factors ($x_t$) from rating class to ratings class. This systematic factor $x_t$can be considered as the aggregated impact of all macroeconomic variables relevant to rating migrations. The goal is now to minimize the residual errors ($\varepsilon_{ij}$) which represent the non-systemic factors or put in other words, one wishes to minimize the part that can't be explained by the common factor in each rating class.

As carefully mentioned above, the next step is to estimate deviations in the transformed 1-year transition matrices ($S_t$) from the long time average ($\bar{S}$) and to extract the part that can be explained by macroeconomic variables ($a_i x_t$) in order to analyze the relation and the sensitivity of those deviations to the state of the economy. To this end the procedure of the BOJ (2009) is followed, assuming that $S_{ij,t}$ can be fitted as the sum of the long time average $\bar{S}_{ij}$ and $a_i x_t$. Subsequently $a_i x_t$ is extracted by minimizing the difference between the observed probability

$$P_{ij,t} = \Phi(S_{ij,t}) - \Phi(S_{ij-1,t}) \tag{10}$$

and the fitted probability

$$\hat{P}_{ij,t} = \Phi(\bar{S}_{ij,t} + a_i x_t) - \Phi(\bar{S}_{ij-1,t} + a_i x_t) \tag{11}$$

by applying the least squares method.

$$\underset{a_i x_t}{Min} \sum_j (p_{ij,t} - \hat{p}_{ij,t})^2 \tag{12}$$

To extract $a_i x_t$ in this manner implies that the shift from the long time average in $S_t$ is of equal magnitude in a particular rating class and by definition the least squares method ensures the minimization of the residual errors ($\varepsilon_{ij,t}$) which represent, as explained above, the non-systematic idiosyncratic factors.

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[16] Where (1-$a_i^2$) is the non-explained variance. Wei (2003) estimates $a_i^2$ by computing $\Delta S_{i,t}$ from the long time average each year for every rating category which results in a time series of $\Delta S_{i,t}$. $a_i^2$ is then approximated by var ($\Delta S_{i,t}$.)

### 2.2.3 Relation to Macroeconomic Variables

Now that the common factor is extracted in the previous section, it is possible to relate this to macroeconomic variables in numerous ways. One could opt for example to fit a polynomial function, use a smoothing spline, or attempt to fit a neural network however keeping in mind goodness of fit criteria[17] and the tradeoff between complexity and predicting power, this paper only considers linear regression with 2 predictors:

$$\alpha_i x_t = c_i + \beta_{i,1} x_{1,t} + \beta_{i,2} x_{2,t} \tag{13}$$

The BOJ (2009) uses real GDP growth and a measure for indebtness as predictors which resulted in an adjusted $R^2$ of 0.7 in the their best 2 out of 3 rating categories and a adjusted $R^2$ of 0.5 in their lowest rating class. Chapter 4 of this paper proves that the choice of different variables outperforms real GDP growth and their measure of Debt as a set of predictors.

Now that a relation between the shift (or deviation) from the long term average transition matrix and the 2 predictor variables is set, one needs to have some indication of the future path of these variables. In fact, these predictions are only a small part of a true stress testing exercise and is commonly referred to as a baseline scenario. In order to really test the resilience of a financial institution one does not only need to test the baseline but has to construct several unexpected scenarios or shocks which are called the adverse scenarios[18]. For example, the EBA sets out adverse scenarios for variables such as GDP growth, inflation, unemployment, asset prices and interest rates and in case of the real GDP growth of the EU as whole, the EBA (2016) applies a growth of 2.0% for 2016, 2.1% in 2017 and 1.7% in 2018 for the baseline scenario compared to -1.2% in 2016, -1.3% in 2017 and 0.7% in 2018 for the adverse scenario.

There exist a lot of potential macroeconomic variables to stress and not all of them can be used as predictors in the simple estimate for $\alpha_i x_t$ and certainly not all of them would be good predictors or even useful to stress, but take for example government bond yield and interest rates as predictors for $\alpha_i x_t$ and the adverse scenario of the EBA: one could assume that a severe shock in GDP would have an impact on interest rates and government bond yield and thus $\alpha_i x_t$. However, a severe shock could also impact other variables that in turn affect interest rates or the government bond yield. These direct and indirect effects can be of variable intensities and can contribute to the same effects, temper or even reverse the impact. Therefore it is important to assess a combined response in interest rates and government bond yield to a shock in GDP. To that end, a VAR model is constructed with the main macroeconomic variables representing the economy.

A VAR model is frequently used in econometrics to capture the relation between several time series. For each time series a linear equation is estimated based on the observed sequence of data points, its own lags as well as the lags of the other time series in the model. A lag of 1 indicates that the value of a time series one unit of time from now not only depends on the current observation but also on the observation the year before and is denoted as VAR(1). Similar a lag of 2 indicates that the value of a time series one

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[17] Such as adjusted $R^2$, variable correlation, RMSE and so on.

[18] In theory one can set up numerous scenarios, some more likely than others. A recent trend is what is called ‘reverse stress-testing’ where a maximal loss is first estimated and then possible scenarios are created which could lead to such severe losses.

unit of time from now depends on the current value as well as the value in the previous two periods and is denoted as VAR(2).[19] VAR models are generally used for forecasting purposes and are very useful as they capture interdependencies among multiple time series. A very useful feature of the VAR models applicable to stress testing is the impulse-response analysis.

In an impulse-response analysis a shock is induced in one of the time series and the response in the other time series to that shock is measured. This response will be the combined effect of the variables in the model. The process is repeated for all the time series in the model in order to analyze the response to various shocks. Clearly, this is exactly the feature one would desire to have in a stress testing exercise. In chapter 4 a VAR model is set up and the impulse-response analysis is carried out.

## 2.3 Capital requirements and Simulation

This section briefly discusses some relevant parts of the Basel frameworks needed to assess and simulate the credit quality portfolio. The Basel frameworks deal with 3 pillars: Pillar 1: Providing means for assessing minimal capital requirements, Pillar 2: Supervisory review which aids regulators to subject financial institutions to proper requirements and Pillar 3: The Market Discipline which aims to increase transparency and disclosure. For this paper deals with the stress testing of credit risk, only these capital requirements outlined in Pillar 1 are dealt with.

### 2.3.1 Capital requirements according to the Basel frameworks

The concepts of capital requirements and capital ratio were first introduced in the Basel I framework of 1988. In that framework, the assets of a bank were grouped into 5 categories and all of these assets were assigned a risk weight in relation to their relative credit risk. After weighting all the assets, a bank was required to hold capital equal to 8% of the total RWAs. Table 1 gives an overview of these asset groups and risk weights.

**Table 1: Overview of the RWAs and asset types according to Basel I (Basel Committee 1988)**

| RISK WEIGHT | ASSET TYPE |
|---|---|
| 0% | CASH, GOLD AND OECD GOVERNMENT OBLIGATIONS |
| 20% | HIGH QUALITY SECURATISATIONS |
| 50% | RESIDENTIAL MORTGAGES |
| 100% | ALL OTHER CLAIMS SUCH AS CORPORATE CLAIMS |

[19] The optimal number of lags depends on the model and the observed data sequence and is determined by applying the Akaike information criterion (AIC) or the Bayesian information criterion (BIC)

In addition some main capital ratios are introduced:

Tier1 Capital Ratio = $\frac{TIER\ 1\ CAPITAL}{TOTAL\ RWA}$ (14)

Capital Ratio[20] = $\frac{TIER\ 1 + TIER\ 2 + TIER\ 3\ CAPITAL}{TOTAL\ RWA}$ (15)

CET1 Ratio[21] = $\frac{CORE\ COMMON\ EQUITY}{TOTAL\ RWA}$ (16)

Although Basel I was revolutionary in seeking a common risk weighted capital standard it did fail to differentiate between low and high quality commercial credits. In addition banks found outlets trough which they could shift their risk exposures Allen L. (2003). To address these issues the Basel Committee proposed new regulation measures in the Basel II framework, published in 2004, which had a last update in 2009. The main contributions of this framework was the introduction of the Standardized and (A)-IRB approach. According to the Standardized Approach risk weights are assigned to exposures relative to their credit rating quality. Table 2 shows these risk weights for corporate exposures:

**Table 2: Overview of the Risk Weights according to Basel II (Basel Committee 2004)**

| CREDIT CLASS | AAA TO AA- | A+ TO A- | BBB+ TO BB- | BELOW BB- | UNRATED |
|---|---|---|---|---|---|
| RISK WEIGHT | 20% | 50% | 100% | 150% | 100% |

This approach favors external credit rating agencies as financial institutions have to rely on their services. In contrast, the (A)-IRB approach differs fundamentally from the Standardized approach: banks are allowed to depart from external credit ratings and compute the PD, EAD and LGD of their individual borrowers using internal models.[22] These parameters are then used as an input in fixed formulae to compute the RWAs disclosed in the Basel II framework (see Appendix 1). Once the RWAs are computed, the minimum Tier 1 capital ratio and CET1 ratio was set at 4% and 2.5% respectively.

[20] Tier1 is core capital and includes equity and disclosed reserves. Tier 2 is supplementary capital including undisclosed reserves subordinated debt and hybrid instruments. Tier 3 capital to cover part of the market risk including more subordinated debt and loss reserves

[21] In comparison with the Tier 1 measure, the Core Common Equity excludes preferred shares and non- controlling interest.

[22] These empirical models need to be approved by local regulators

The measures proposed in the Basel II framework clearly did not suffice as it could not prevent the severe financial crisis in 2007-2009. As a response, the Basel Committee on Banking Supervision revised their standards in the Basel III framework which was published in 2010. This revision resulted in an increase of the minimum CET1 ratio from 2.5% up to 4.5% and an increase of the minimum Tier 1 ratio from 4% up to 6%. In addition the Basel III framework introduces the leverage ratio:

$$\text{Leverage Ratio} = \frac{TIER\ 1\ CAPITAL}{TOTAL\ EXPOSURE} \tag{17}$$

This measure is the Tier 1 capital divided by the non-risk weighted exposure and the minimum is set at 3%. Finally, Basel IV is set to be published in 2019 however the main objectives are already made public: a further increase in capital requirements, a distancing from the (A)-IRB approach for increased comparability and a revision of the risk weights

### 2.3.2 Simulation

The capital requirements reviewed in the previous section can be used as a benchmark of portfolio quality. After all, these measures are the minimal capital requirements and thus if a simulation of a bank's portfolio results in ratios below the minima the bank does not prove to be resilient. This simulation completes the framework outlined in section 2.2. The last step discussed in that section consisted of constructing baseline and adverse scenarios and setting up a VAR model in order to perform an impulse-response analysis which eventually provides input for eq. (13). A stress test is typically applied to 3 subsequent years[23] which means that $\alpha_i x_t$ needs to be assessed in the 3 years following the year of the stress test. Once $\alpha_i x_t$ is obtained for $t = 1..3$ the shift is applied to the long term average $\bar{S}$ and the whole process outlined in 2.2.1 is reversed to obtain the implied transition matrices. Starting from an initial portfolio a bank can now simulate the credit migrations according to the implied transition matrices in year $t = 1..3$ and can compute the implied RWAs according to the (A)-IRB or the Standardized approach. Chapter 4 performs such a simulation on a sample portfolio.

[23] Because making predictions after year 3 is highly uncertain.

# Part 2: Results

# 3 Data

As is presumably the case with all large datasets, this paper has to deal with data issues. These problems as well as the approach for dealing with them are discussed in section 2 of this chapter.

## 3.1 Data selection

The sample in this study starts in December 1995 and runs to December 2014. All data (macroeconomic, accounting and market) are retrieved from Thomas Reuters' Datastream. The focus of this study is on the corporates in the Eurozone (excluding financial services). This leads to a sample of 5361 companies, including large corporates as well as SMEs, both active and defaulted corporates, in a variety of industries. The sample is dynamic meaning that corporates can enter and leave the sample each year. The 3 models described in section 1.3 are applied resulting in the collection of yearly, quarterly as well as monthly data depending on the variables used in those scoring models. For a more detailed overview of the composition of this sample see Appendix 2.

The time series of macroeconomic variables are obtained from Thomas Reuters' database as well. This data originates from the EUROSTAT and ECB databases and applies to the Eurozone as a whole and include typical variables reflecting output such as : Real GDP, Sales index and Trade Balance ; rates such as: Interest Rates, Government Bond Yield , Effective Exchange Rates and Unemployment rate ; Inflation measures such as : HICP, CPI and the GDP Deflator. The time series are quarterly series and where needed are converted to yearly series.

## 3.2 Data Problems

Throughout the paper there is a constant confrontation with data problems. A clear perspective has to be taken. The objective set, is to manipulate the data as less as possible resulting in the omission of traditional methods such as interpolation and extrapolation.

### 3.2.1 Missing data

As the empirical period spans from 1995-2014 missing data can occur due to delisting, defaulting or simply due to lack of reporting. The total sample of corporates considered is divided in 2 samples with companies that are still active and those who are not, referred to as the 'active sample' and 'the defaulted sample', respectively. In the active sample 3 events are distinguished. A company can miss data in 1 particular year which is referred to as 'Single gap', in 2 consecutive years referred to as 'Double gap' and in 3 or more years. Using traditional techniques as interpolation, extrapolation or cross sectional mean for dealing with this issue is not advised because of the great number of variables used to compute the credit score. Interpolation for example would require to interpolate, in the year(s) data is missing, in each and every variable (because they are all missing). The risk of creating outliers instead of reducing them is therefore increased significantly as those variables are used as input to one of the credit scoring models and thus the possible error created by interpolating is magnified.

This paper proposes a different approach: if the data required to compute the score is missing in at least 1 of the variables, no score is computed in that year and thus no rating is assigned. Next the Single gap and the Double gap is filled with the help of pattern recognition with the most likely rating. For the active sample: if data is missing in more than 2 consecutive years, the company is treated as a new company as the most likely reason for the missing data is the delisting and relisting of the company and to do predictions 3 years ahead is very uncertain. For the defaulted sample, the matter is a little more complicated. Thomas Reuters provides the legal date of default, however in about 10% of the cases in that sample, data is missing in the 3 consecutive years prior to default. Because these companies still contain information about the credit rating migration up to the date the data is missing they are left in the sample. For defaulted companies with missing data in less than 3 consecutive years prior to default, the same procedure is executed as in the case of the active sample. Table 3 provides an overview.

**Table 3: Overview of the number of gaps in the sample (Vedran Cengic 2016)**

| TYPE OF GAP | SINGLE GAP | DOUBLE GAP | 3 OR MORE |
|---|---|---|---|
| TOTAL | 85 | 146 | 27 |

Keeping in mind that a sample of 5361 over a span of 19 years results in a matrix of 101 859 elements (5361 x 19) one can conclude that gaps represent only a small part of the sample[24]. To fill these gaps neural networking is applied and in particular its application to pattern recognition. These neural networks are inspired on the way the human brain works. Although the idea is not new, recent developments and increase in computational power have opened the window for numerous applications: from robotics, classifications and imaging to this paper. To fill in the gaps, first the transition matrices are set up leaving the gaps empty. Next, the whole framework described in 2.1 is executed and after obtaining the deviations and regressing to the macroeconomic variables a selection of preferable variables is made. These variables are then used as an input in the neural network along with the current credit rating in year t and the credit rating in the previous year t-1 for all the companies in the sample in each year. Thereafter the neural network trains for a predefined number of times in order to learn the most likely credit migration paths in the sample. Finally when the neural network is done training, the gaps are filled with their most likely rating relative to the current macroeconomic state and their observed credit migrations so far. For a more detailed explanation as well as theory on neural networking and their application to this paper see Appendix 3.

[24] An attentive reader will notice that not all of the 101 859 elements in the matrix are filled. A large part of the sample for example did not even exist in 1995. It is difficult to assess how much of the matrix is exactly filled. In any case the gaps represent only a fraction of the sample.

### 3.2.2 Outliers

Missing data was not the only problem. Due to the size of the sample it was unavoidable to have some outliers which threaten the stability of the rating system. Just as the problem with the missing data, a clear perspective has to be taken and the main issue of outliers in this paper is illustrated below.

Suppose that no missing data exists and that all companies in the sample exist over the entire period, one would have to check 101 859 data points for every variable for outliers. This would clearly be an inefficient and very time consuming exercise. Instead, all the PDs of the companies are computed based on one of the credit scoring models. Very high PDs or very low PDs affect the DE-Algorithm and the eventual thresholds of the distinct rating categories[25]. Therefore, the cumulative distribution of all the PDs computed in a particular year are fitted, in order to extract the values at the 5th and the 95th percentile. PDs lower than the 5th percentile are replaced with the value at the 5th percentile, and similar for the PDs above the 95th percentile. If the 5th percentile turns out to be less than 3 basepoints, the PDs will be capped at this level. This was however never the case.

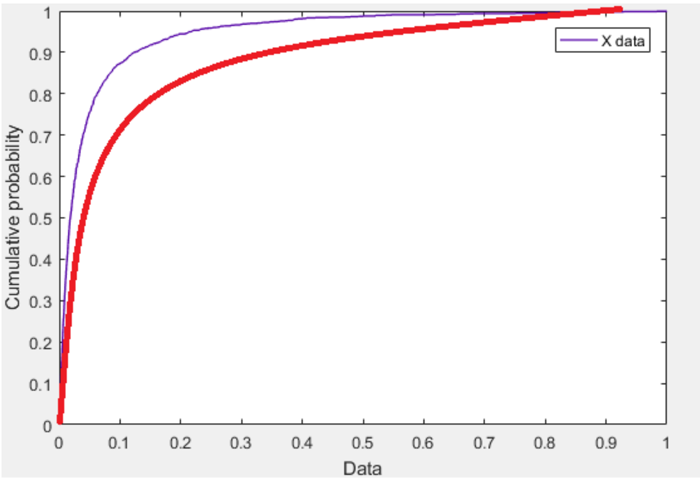


**Figure 4: Observed and reasonably fitted cumulative PDs (Vedran Cengic 2016)**

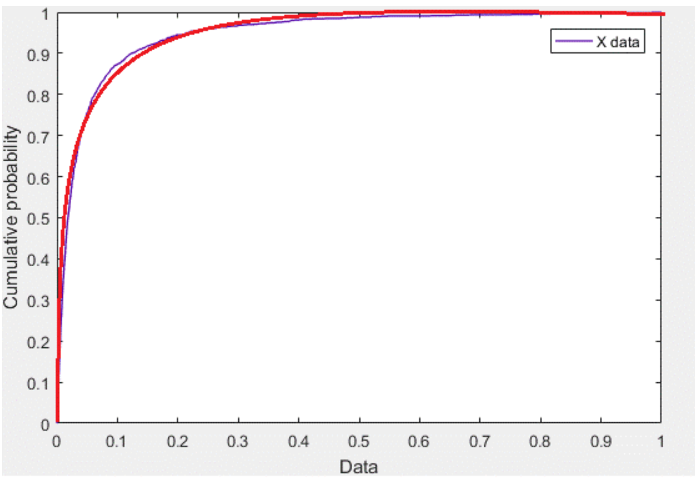


**Figure 5: Observed and nearly perfect fitted cumulative PDs (Vedran Cengic 2016)**

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[25] The wider the range of the PDs the wider the thresholds.

Figure 4 represents a reasonable fit of the cumulative probabilities of the observed PDs in the sample of a particular year. The blue line represents the cumulative probabilities of the observed PDs obtained from applying one of the credit scoring models and the red line represents the fitted distribution. One can conclude from Figure 4 that about 90% of the sample during a particular year have a PD between 0 and 0.1 and about 3% have a PD between 0.5 and 1. As these are very high PDs, they are probably representing 3% of the sample that are outliers. At first glance this seems to be a reasonable good fit. However when the values are capped at the 95th percentile, analyzing the blue line would result in PDs between [0, 0.2] whilst analyzing the red line representing the fitted distribution would result to values between [0, 0.7]. It is clear from this example that this would have a major impact on the DE- algorithm and eventually on the stability of the rating system. Finding a perfect (or nearly perfect) fit is thus of high importance. Figure 5 illustrates a near perfect fit which is obtained using the non-parametric Kernel distribution. Overfitting is usually a problem when predictions have to be made, however as this is not the case here, overfitting is even welcomed. This is the main impact outliers can have in the stress testing exercise as the construction of the transition matrices is a key element in the framework. Further discovery and handling of outliers can be performed by the help of neural networks as they can help detect very unlikely behavior in credit migrations. This however, will not be illustrated in this paper and could be the subject of future research.

Following this procedure resulted in a much more stable rating system. Not only does the DE-algorithm need less effort to find a (sub) optimal solution but the transition matrices as illustrated in the next section show increased concentration on the diagonal which is the desired effect.

# 4 Results

This chapter discusses the main results obtained by chronologically executing the framework set out in 2.1

## 4.1 Transition Matrices

According to the first step in the framework in 2.1, the PDs have to be computed using one of the credit scoring models, however in order to compare, the PDs are calculated using all the models. The next step is to bucket these PDs to find the pooled PDs using the DE-algorithm. It should be clear by now that the DE—algorithm is a very important part of the framework. Which is why it was subjected to extensive testing. Different settings for the parameters are evaluated based on their output in the objective function eq. (2). Eventually no significant improvement is found after 1000 iterations and a generation of 1000 candidate solutions in each iteration. At this number of iterations, no essential difference is observed between the 3 strategies in terms of their solution of eq. (2) however DE/BEST/1 converges much faster than the DE/RAND-TO-BEST/1 and slightly faster than the DE/RAND/1 strategy. Similarly, different settings for the CR resulted in worse performance for lower values of CR: the convergence to the optimal solution was much slower and less accurate, consequently the CR parameter is set at 0.9 meaning that with a probability of 90%, the DE/BEST/1 strategy will be exercised. After setting these parameters the algorithm is run every year during the sample period and ratings are assigned to each individual borrower based on the computed thresholds of the rating classes in each year. The first test run turned out to be somewhat unstable, especially in the CCC rating class where the total number of CCC rated companies varied significantly each year from only 3 CCC rated companies in a particular year to over 300. This is not a desired behavior of a stable rating system and to deal with this issue a severe penalty is added to the algorithm which adds a significant cost each time in a proposed solution a rating class contains less than 5% of the total sample of companies in a particular year. After this adjustment, the transition matrices were much more stable. Figure 6, 7 and 8 show the long time average transition matrices based on the Z-Score, the O-Score and the CHS model respectively. For an example of the S&P long term average transition matrix see Appendix 4.

**Figure 6: Long time average transition matrix: Z-Score (Vedran Cengic 2016)**

| | AAA | AA | A | BBB | BB | B | CCC,CC,C |
|---|---|---|---|---|---|---|---|
| AAA | 87.13 | 10.15 | 1.94 | 0.46 | 0.13 | 0.10 | 0.09 |
| AA | 8.37 | 78.55 | 9.80 | 2.30 | 0.64 | 0.23 | 0.12 |
| A | 1.14 | 10.52 | 74.61 | 10.10 | 2.55 | 0.78 | 0.30 |
| BBB | 0.44 | 2.24 | 11.93 | 72.14 | 10.07 | 2.54 | 0.64 |
| BB | 0.15 | 0.76 | 2.85 | 12.01 | 72.39 | 9.90 | 1.94 |
| B | 0.09 | 0.38 | 0.82 | 3.11 | 12.46 | 75.04 | 8.11 |
| CCC,CC,C | 0.13 | 0.15 | 0.55 | 1.09 | 2.94 | 11.01 | 84.13 |

**Figure 7: Long time average transition matrix: O-Score (Vedran Cengic 2016)**

| | AAA | AA | A | BBB | BB | B | CCC,CC,C |
|---|---|---|---|---|---|---|---|
| AAA | 83.85 | 10.74 | 2.90 | 1.08 | 0.53 | 0.24 | 0.66 |
| AA | 10.42 | 74.25 | 10.25 | 2.93 | 1.02 | 0.48 | 0.64 |
| A | 3.97 | 12.88 | 68.90 | 8.27 | 3.37 | 1.37 | 1.24 |
| BBB | 2.46 | 5.89 | 12.91 | 65.78 | 6.99 | 3.59 | 2.38 |
| BB | 1.64 | 3.29 | 7.76 | 9.43 | 65.74 | 7.25 | 4.89 |
| B | 1.55 | 2.34 | 4.87 | 6.42 | 8.37 | 67.52 | 8.93 |
| CCC,CC,C | 2.01 | 1.78 | 2.74 | 3.14 | 3.56 | 4.96 | 81.81 |

**Figure 8: Long time average transition matrix: CHS-Model (Vedran Cengic 2016)**

| | AAA | AA | A | BBB | BB | B | CCC,CC,C |
|---|---|---|---|---|---|---|---|
| AAA | 72.04 | 13.86 | 7.23 | 2.97 | 1.53 | 1.11 | 1.26 |
| AA | 14.47 | 62.17 | 11.60 | 4.93 | 2.82 | 2.03 | 1.98 |
| A | 9.73 | 13.41 | 60.80 | 6.38 | 3.89 | 2.68 | 3.12 |
| BBB | 7.06 | 11.15 | 12.83 | 55.60 | 5.12 | 3.97 | 4.28 |
| BB | 5.89 | 9.58 | 11.63 | 7.76 | 54.45 | 4.07 | 6.62 |
| B | 5.49 | 8.11 | 10.80 | 7.39 | 5.87 | 55.04 | 7.31 |
| CCC,CC,C | 4.75 | 6.54 | 8.03 | 5.65 | 5.03 | 4.66 | 65.34 |

What immediately attracts the attention in all of these transition matrices is the dominant diagonal which is desired in a TTC approach as explained in 1.1. Furthermore, this diagonal is much more prominently present in the first transition matrix and especially compared to the last matrix. In fact, the last transition matrix presents relatively high transition probabilities off of the diagonal and is the least stable of the 3 models considered. The issue of stability of the transition matrix in the CHS- model does not prove that the CHS-model is a bad model but it does however prove the CHS-model to be less applicable for creating transition matrices. This is due to the high required quarterly and monthly data that is needed to compute the PDs. Lots of companies however do not disclose quarterly data. Note in addition that migrations to the default state are not considered for two reasons. Firstly, the quality of the exact date of default is not reliable due to a lot of missing data in the years before the legal default date and second, the pooled PDs are already obtained in the previous step by means of the DE-algorithm.

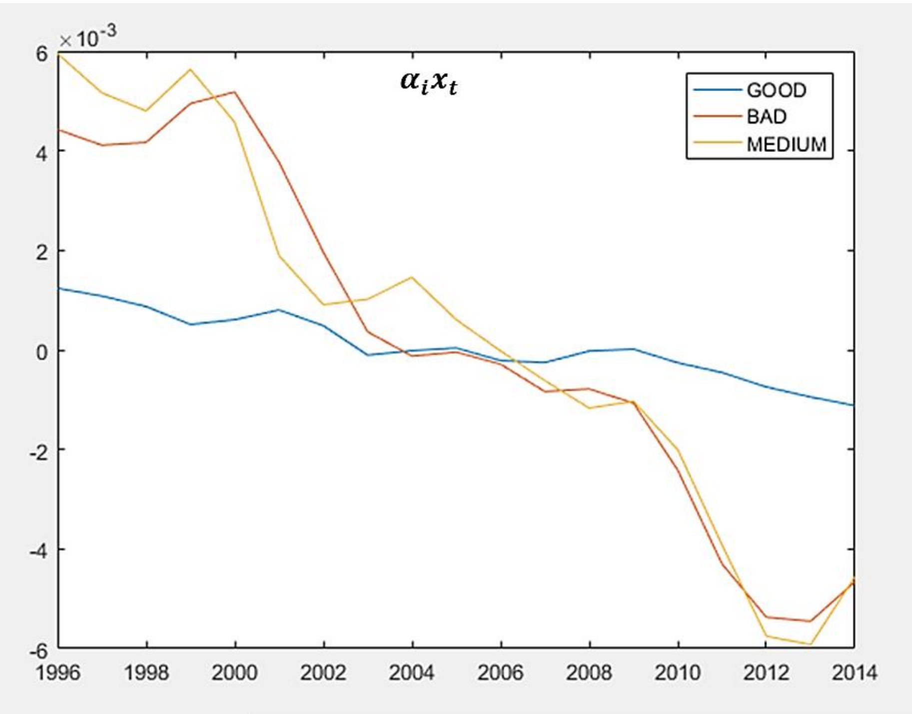
## 4.2 Measuring deviations and link to macroeconomic variables

The next step in the framework was to compute deviations from the long time average. As explained in 2.2.1 the log logistics distribution is used instead of the normal distribution due to its better fit. As an example, the transformed long term transition matrix for the O-score is showed below and the remainder of this paper will illustrate the framework using the transition matrix based on the O-score as it showed the best fit to the macroeconomic variables.

$$\bar{S} = \begin{pmatrix} 0,005305 & 0,001759 & 0,000807 & 0,000378 & 0,000214 & 0,000152 \\ 0,272157 & 0,005453 & 0,001636 & 0,000527 & 0,000227 & 0,000123 \\ 0,554218 & 0,184446 & 0,015444 & 0,005978 & 0,002884 & 0,001705 \\ 0,844032 & 0,342226 & 0,154792 & 0,012646 & 0,005756 & 0,002875 \\ 1,111912 & 0,538985 & 0,274227 & 0,1331 & 0,012022 & 0,00567 \\ 0,954444 & 0,518747 & 0,292987 & 0,161936 & 0,098837 & 0,008114 \\ 0,756998 & 0,499813 & 0,353064 & 0,228852 & 0,166089 & 0,124405 \end{pmatrix} \quad (18)$$

After obtaining eq.(18) it is now possible to measure the deviations from that average. When computing these deviations, some unusual patterns were observed particularly in the CCC rating class. To address these unusual patterns, the rating classes are grouped into 3 groups. {(AAA, AA), (A, BBB, BB), (B, CCC)} and deviations from the average of these 3 groups are measured. This approach resulted in much more reasonable result. Figure 9 illustrates the deviations from the long term average for the O-score of these 3 groups.

**Figure 9: Deviations from the long term average (Vedran Cengic 2016)**




From Figure 9 one can conclude that in the period 1996-1999 $\alpha_i x_t$ increases to above average values, decreases after 2000 and drops severely after 2008 with signs of slow recovery after 2013. These results correspond to the empirical observations: for example the bubble burst in 2000 and the recent severe financial crisis in 2008. Furthermore, the figure suggests that the high rated companies are less affected by business cycles than the lower rated companies and that the medium group reacts in a similar fashion as the lower rated group.

The next step is to regress these deviations to macroeconomic variables. In order to select the best fitting model the following approach is followed:

1. The macroeconomic variables are made stationary by taking the log and then differencing. Rates are left unchanged.
2. The deviations are regressed to all possible combinations of 2 variables out of the sample
3. Scores are assigned to the models with the lowest score indicating best fit[26]
4. The models are then sorted by their scores with the lowest scores on top

Following this approach 2 interesting models are selected and their estimations are showed in table 4 and 5 below for the first rating group. For the estimates of the 2 other rating groups see Appendix 5.

**Table 4: Model 1 GOOD rating group (Vedran Cengic 2016)**

| | CORRELATION | | | ESTIMATES | P-VALUE | ADJ. $R^2$ |
|---|---|---|---|---|---|---|
| CONSTANT | 1 | -0.97 | 0.093 | -0,0022 | $2.68X10^{-7}$ | **0.828** |
| G. BOND YIELD | -0.968 | 1 | -0.16 | 0,0005 | $1.49X10^{-7}$ | |
| SALES[27] | 0.093 | -0.16 | 1 | 0,0059 | 0.0892 | |

**Table 5: Model 2 GOOD rating group (Vedran Cengic 2016)**

| | CORRELATION | | | ESTIMATES | P-VALUE | ADJ. $R^2$ |
|---|---|---|---|---|---|---|
| CONSTANT | 1 | -0.96 | -0.05 | -0.0023 | $1.85X10^{-7}$ | **0.826** |
| G. BOND YIELD | -0.958 | 1 | -0.11 | 0.000526 | $1.25X10^{-7}$ | |
| GDP DEF | -0.045 | -0.11 | 1 | 0.0058 | 0.0982 | |

Both models include predictors that are very significant according to their P-values. Furthermore both models produce a very high adjusted $R^2$ for the GOOD rating group and around 0.7 for the other rating groups (see Appendix 5), however the first model outperforms the second model in the other 2 rating groups by approximately 5%. In addition, the choice for sales output as a predictor is intuitively more attractive as this is considered a very good economic indicator. The signs are expected and confirmed to be

[26] For example a very low correlation earns one point, the highest correlation earns 10 points. Similarly High adjusted $R^2$ earns 1 point whereas very low adjusted $R^2$ earns 10 points.

[27] Retail Sales Volume index for the whole Euro zone.

positive for both the 10- year government bond yield and the sales. The explanation for the positive sign of sales is somewhat trivial as more sales indicate better economic conditions and increased consumer confidence. For the intuition behind the positive sign of the 10-year-government bond yield an illustration of the historic evolution is helpful.

**Figure 10: YoY changes government bond yield 1996-2014 (Vedran Cengic 2016)**

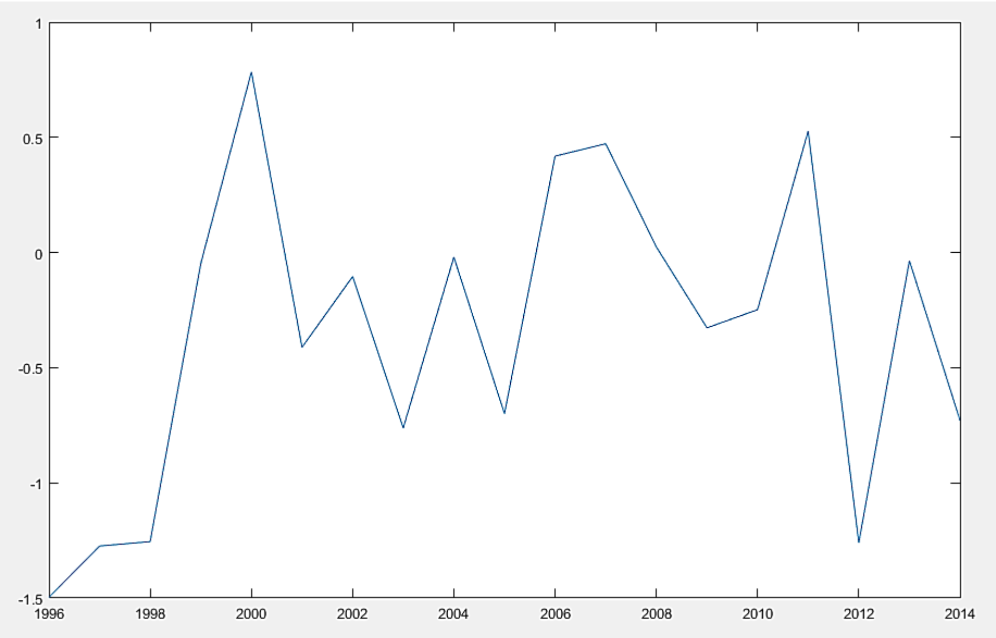



Figure 10 clearly shows that in times of economic expansion (1996-2000) government bond yields go up and in times of recession yields go down with 2000 and 2008 as clear examples. These bonds thus tend to signal investor confidence. When confidence is high, the 10-year treasury bond's price drops and yields go up because investors feel they can safely invest somewhere else at higher rates of return, but when confidence is low, the price goes up as there is more demand for this safe investment, and yields fall.

The signs of the second model are also expected and confirmed to be positive. The 10-year government bond yield is explained above, but the GDP deflator is less clear as a predictor compared to sales and is an additional reason for preferring the first model over the second model. The GDP deflator is measured by:

$$GDP\ DEFLATOR = \frac{NOMINAL\ GDP}{REAL\ GDP}\ x\ 100 \qquad (19)$$

and is a measure of inflation. A positive sign thus indicates that economic growth is positively related to higher inflation and vice versa in the case of recessions. There are several reasons why economic growth can lead to increased inflation. If for example demand increases faster than companies can supply, companies respond by increasing prices. Furthermore a rapid growth can increase the labor demand and thus lead to an increase in wages these effects lead to a higher inflation. Although economic growth can stimulate inflation, it can also have no effect (or slight) on inflation. This is the case if growth arises from an increase in productivity which can handle demand. For this reason model 1 is preferred over model 2 as both predictors have straightforward impact.

## 4.3 VAR model and impulse-response analysis

Now that a model is chosen, following the framework, a VAR model has to be set up and scenarios need to be constructed. This paper considers a shock in real GDP growth. In order to analyze the responses of the government bond yield and the sales, the VAR model must at least consist out of these 4 variables. In addition 2 variables are included in the VAR model as they are hypothized to have an important impact on the economy: the GDP deflator as a measure of inflation and the effective exchange rate as an indicator of monetary strength. Similar to the linear regression, all these variables (except the rates) are made stationary by taking the log and then differencing. See Appendix 6 for the stationary evolution of these variables over the 80 quarters (1995-2014). Next the optimal lag is determined through the AIC criterion[28] and a maximum of 5 lags is considered[29] :

**Table 6: Selecting the optimal lag (Vedran Cengic 2016)**

| LAGS | AIC |
|---|---|
| 1 | -3410.56 |
| 2 | -3419.55 |
| 3 | -3411.6 |
| 4 | -3443.2 |
| 5 | -3425.12 |

The optimal model for these sets of variables is thus a model with 4 lags or 4 quarters VAR (4). After the model is estimated (see appendix 7 for the estimates of the VAR (4) model), the next step is to assess forecasting quality. This is done by calibrating the model up to 2010 and the period of 2010-2014 is used as a test sample after which a Monte Carlo simulation of 100 000 simulations is then applied to this test sample. The forecasting quality is then assessed by measuring the percentage RMSE[30]. See Appendix 8 for an overview of the percentage RMSEs for the different variables. Overall the model performs very well and succeeds in doing reasonable predictions as the maximum mean percentage RMSE is only about 6% which is very low.

[28] The AIC criterion provides a way for model selection and a lower value indicates a better model.

[29] Recall that x lags indicate dependency of x previous periods. Obviously the more lags are implemented the more complex the model gets.

[30] Interpret this as being 6 % off the actual value. The reason for choosing percentage RMSE is because the variables have different scales. The scale of Government bond yields for example is a factor of 10 higher as it is not log differenced.

### 4.3.1 Scenarios

Now that it is confirmed the model performs reasonably well, some forecasts can be done in order to construct the baseline scenario.

**Figure 11: Forecast of sales (Vedran Cengic 2016)**

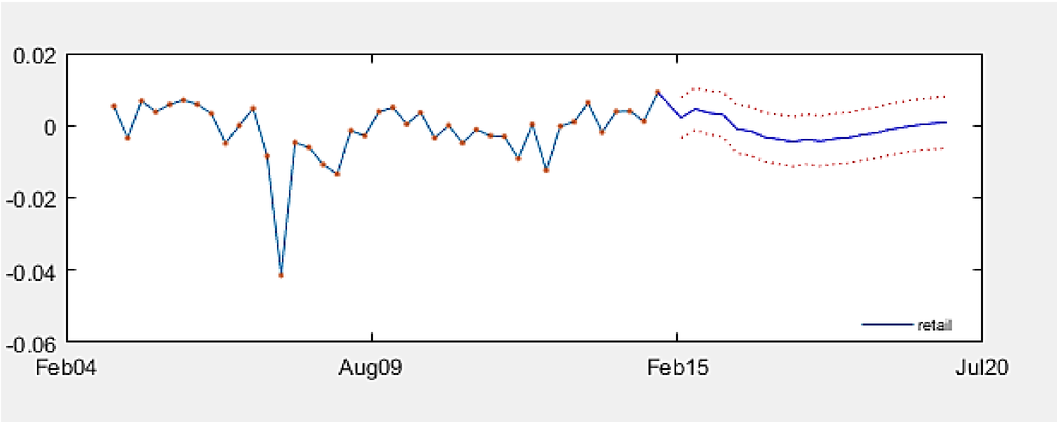



**Figure 12: Forecast of the 10-year government bond yield (Vedran Cengic 2016)**

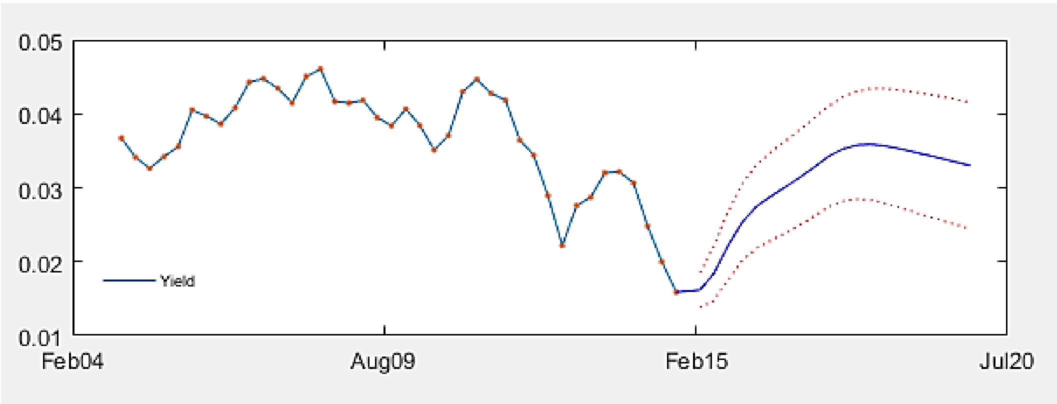



Figure 11 depicts the sales forecast and figure 12 the forecast of the government bond yield (see Appendix 9 for forecasts of the other variables). Both figures display the historic pattern (from 2004-2014) as well as a forecast for the period (2015-2020). Figure 11[31] suggests that the sales will go down in 2015 after which it will stabilize. The red dotted lines represent the 95% confidence bounds and in the case of the sales forecast, the model is quite confident. In Figure 12 an upwards trend after 2015 is suggested by the model and as we now experience historically very low yields, the model suggests a relatively strong recovery impulse in 2015 and 2016 after which it will slowly decay. Looking at the confidence levels, one can conclude the model is less confident of this prediction.

[31] Keep in mind that the sales output is log differenced.

Next 5 units of shock are applied to the real GDP growth[32]. The equivalent size of these shocks in each year as well a comparison with the EBA 2014 adverse scenarios of the real GDP is given in table 7.

**Table 7: Comparison of baseline and adverse GDP scenarios (EBA 2014, Vedran Cengic 2016)**

| | EBA(%) | | VAR(4)(%) | |
|---|---|---|---|---|
| YEAR | GDP B | GDP ADV. | GDP B | GDP ADV. |
| 2015 | 2.0 | -1.5 | 2.5 | -1.66 |
| 2016 | 1.8 | 0.1 | 2.24 | 0.15 |
| 2017[33] | / | / | 0.34 | 0.78 |

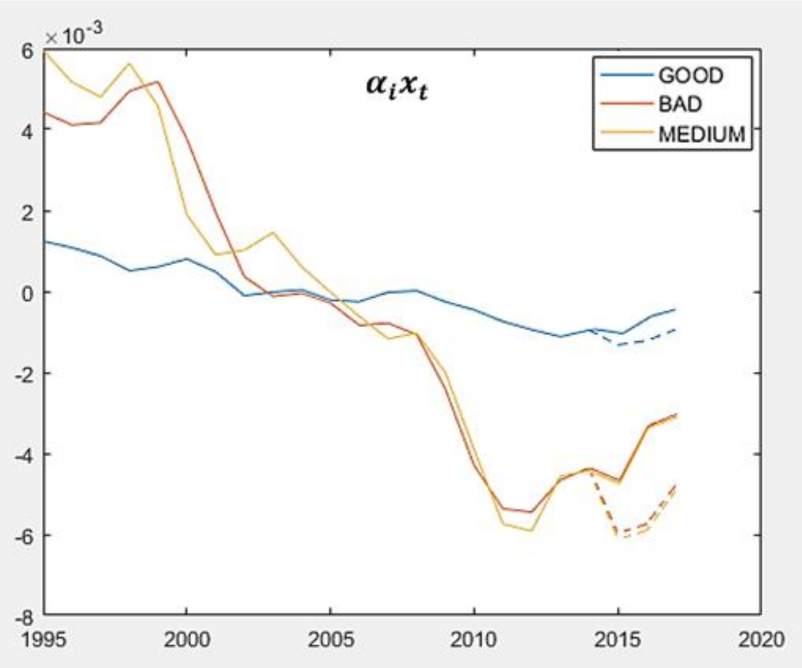
Compared to the EBA stress test, the VAR(4) model predicts a slightly higher GDP growth in 2015 and 2016 however this growth is predicted to stagnate in 2017 ( See Appendix 10 for the real GDP projection) according to the VAR(4) model. The adverse scenarios are similar in both models although the VAR(4) model suggest a slightly larger shock and faster recovery. Appendix 11 illustrates the responses to shocks in the macroeconomic variables included in the VAR(4) model. The shocks are presented on the vertical axis and the responses on the horizontal axis. One can conclude that a shock in real GDP (first row), not surprisingly leads to a drop in sales. Interesting to see is that the sales react very positive to signs of GDP recovery. Similarly the 10-year government bond yield drops about 1% relative to the baseline as a response to the shock, and reacts less enthusiastic to signs of GDP recovery compared to the sales. This result corresponds to the discussion of Figure 10 and the VAR(4) model again proves to predict reasonable outcomes. Once these responses are obtained and analyzed, they can serve as an input to the model estimated in eq. (13). Figure 13 illustrates the implied deviations according to the baseline (full line) and the real GDP shock (dotted line). According to this figure, the baseline scenario implies a recovery resulting in a decrease of the deviations from the long time average. As expected the shock in GDP implies increasing deviations in 2015 and 2016 after which it gently recovers. Similarly to the discussion of Figure 9, the shock in GDP affects the high rated group less severe than the lower rated groups.

The final step before heading to the simulation is to reconstruct the implied transition matrices by applying the obtained deviations to shift the long time average S-Matrix ($\bar{\bar{S}}$) and by reversing the process outlined in section 2.2.1. Appendix 12 and 13 show these implied transition matrices for the baseline and the adverse scenario respectively.

[32] Other shocks can be considered as well, however this paper will only focus on a shock in GDP growth.

[33] The EBA stress test in 2014 includes the years (2014, 2015 and 2016) whereas this exercise ranges from 2015-2017

**Figure 13: Deviations from the long time average according to the baseline and adverse scenarios (Vedran Cengic 2016)**




When comparing these implied transition matrices with the long time average, one can immediately notice less stable transition matrices as the concentration of the transition probabilities on the diagonal of both these implied transition matrices is lower than the long time average transition matrix. In addition one can notice in each rating category of these implied transition matrix a shift towards the CCC, CC, C ratings. Both matrices show less stable behavior in the first year and a slight recovery by the end of year 3. Furthermore, the transition matrix of the baseline scenario shows more stable behavior than the adverse scenario and is relatively close to the long term average which corresponds with Figure 13.

### 4.3.2 Simulation

The final step is to simulate the credit migrations of a portfolio of companies based on the implied transition matrices acquired above and to compute the RWAs in order to obtain the CET1 ratios eq. (16). This is done following the Standardized Approach with the risk weights as in table 2. Appendix 14 illustrates the simulated CET1 ratios in 2015, 2016 and 2017 for the baseline and adverse scenario as well as the CET1 ratios based on the long time average transition matrix. For this simulation, the same portfolio is used for all the scenarios and this portfolio consists of 500 companies randomly picked out of the whole sample of 5361 companies. Table 8 shows the composition of the portfolio.

**Table 8: Rating composition of the portfolio considered (%) (Vedran Cengic 2016)**

| RATING | AAA | AA | A | BBB | BB | B | CCC;CC;C |
|---|---|---|---|---|---|---|---|
| % | 5 | 8 | 24 | 30 | 15 | 10 | 8 |

As stated in section 2.3.1 the minimum CET1 ratio following the Basel III framework is 4.5%. With an initial CET1 ratio of 11%, the initial capital held is not adequate in the case of the adverse scenario as the CET1 ratio falls below 4.5% to 3.25%. In the baseline scenario, the capital requirements are satisfied, although not comfortably. Only when applying the long time average transition matrix the capital requirements are met with flying colors. This is of course because there is no shock and migrations to lower rating classes such as the CCC, CC, C ratings are low.

Looking at eq. (16) one could conclude that a bank can increase their CET1 ratio by simply increasing the common equity (numerator). However, having high capital requirements is not desirable as it increases the cost of capital and forces banks to cut lending and to increase their interest rates as a compensation which has a direct impact on the profitability. It is thus in a banks best interest to find means to assess high quality portfolios in order to minimize the capital requirements. However, setting the common equity too low can cause a bank to fail meeting its capital requirements as is the case in the adverse scenario. Here a shock in GDP has a negative effect on the sales output because consumers and suppliers are less confident in the economy. This leads to investors buying more government bonds as they are now uncertain to invest in other projects with higher returns which in turn leads to an increase in prices of these bonds and consequently a drop in their yields. Because sales drop, companies struggle as they rely on their sales to remain solvable and thus fail to repay their loans to the bank. As more and more companies default, banks have to cover higher losses with common equity. In addition more companies migrate to lower rating classes and assuming the Standardized Approach this leads to applying higher risk weights to their exposures (see table 2) which leads to an increase in RWAs and following eq. (16) eventually results in lower CET1 ratios. When customers notice that their bank fails to meet capital requirements, especially in a downturn state of the economy, it could cause a bank run where a mass of customers starts withdrawing deposits from the bank. This could destabilize the bank even further and cause it to go bankrupt.

In the case of the long term transition matrix, no shock was observed and thus lower losses are acquired, meaning that the common equity remains high and migrations to the CCC,CC,C ratings remain low. For example in year one, only 0.66% of AAA rated companies will migrate to CCC,CC,C ratings. The result is a higher CET1 ratio. Similar for the case of the baseline scenario. Only now, more migrations occur to the lowest rating class (4%). Because of the higher pooled PD in this rating class, more companies are likely to default. This results in more acquired losses covered by the common equity and because more migrations occur to lower rating classes the RWAs increase resulting in a lower CET1 ratio relative to the long term average. .

# General Conclusion

Performing a macro stress test is a very challenging exercise, especially because one has to consider many aspects, all of which are highly interconnected. Assumptions have to be handled with care as well as data. The last is of particular importance as the financial world is not known for their transparency. This leads to an abandonment of the CHS-model early on in the process as it relies upon quarterly data as well as monthly which results in a lot of missing data and unreliable results.

This paper described the framework for macro stress testing credit risk and the assessment of credit portfolio quality. It started with the construction of the transition matrices with the help of the DE-algorithm. In addition, this paper provides insight on the importance of the choice of the algorithm as the framework is highly interlinked and builds upon these transition matrices. Only 7 rating categories are considered in this paper even though the DE-algorithm is very flexible and can accommodate multiples of rating classes. It could be the subject of further research to attempt and find the optimal number of rating categories.

The MF model proves to be a flexible model, able to capture the main credit migrations observed in recent years. An interesting feature of this model is that it can model downgrades as well as upgrades and different sensitivities of the rating categories to macroeconomic shocks. Furthermore it provides a clear way to link the observed deviations to macroeconomic variables. This paper uses the log logistic cumulative distribution fitting in every rating category. Although this is an improvement relative to the normal distribution used by Wei (2003) further improvements would be welcomed in order to produce smoother results. In addition, the rating classes are grouped in 3 groups, as extracting the common factor out of every rating category produced unreliable results. The grouping of the rating classes is done in a somewhat arbitrary way and different groupings can be considered.

This paper also provides means for dealing with missing data and although it was not the primary focus, neural networking and in particular pattern recognition prove to be very promising methods. As was already hinted, patter recognition could also be used to detect outliers or unusual credit migration behavior and can serve as a very interesting research topic.

Finally, a random portfolio is simulated and the impact on the quality due to macroeconomic shocks are assessed. This paper only considers a shock in real GDP for its comparability with the EBA EU wide stress test. Evidently, multiple scenarios can be constructed such as a shock in interest rates or exchange rates and in particular a framework for integrating reverse stress testing can be further investigated.

Although this framework has a lot of strong elements, its main drawbacks are the sensitivity to the chosen distributions, and the use of the pooled PDs. These pooled PDs are kept static in the computation of the RWAs and a major improvement would be to not only measure the deviations of the yearly transition matrices from the long time average, but to also capture the dynamics in the pooled PDs. This way, when stress testing, implied transition matrices as well as their pooled PDs can be constructed.

Finally this paper discussed briefly the main elements of the Basel frameworks relevant to credit risk and outlined its presumed future path

# Appendices

## ***Appendix 1: Formulae for assessing RWA in the A-IRB approach***

Below the formulae for computing the RWAs for corporate exposure are shown (Basel 2004)

- N(x) = Normal cumulative distribution function
- G(x) = Inverse normal cumulative distribution function
- PD = Probability of default
- LGD = Loss given default
- EAD = Exposure at default
- M = Effective maturity

**Correlation**

$$R = AVC * \left[0.12 * \frac{1 - \ e^{-50+PD}}{1 - e^{-50}} + 0.24 * \left(1 - \frac{1 - e^{-50+PD}}{1 - e^{-50}}\right)\right]$$

with AVC = 1.25 if the company is an unregulated financial institution and AVC=1 in all other cases.

**Maturity adjustment**

$$b = (0.11852 - 0.05478 * \ \ln(PD)^2$$

**Capital requirement**

$$K = LGD * \left[N\left(\sqrt{\frac{1}{1-R}} * G(PD) + \sqrt{\frac{R}{1-R}} * G(0.999)\right) - PD\right] * \frac{1 + (M - 2.5)b}{1 - 1.5b}$$

$$RWA = K * 12.5 * EAD$$

## *Appendix 2: Composition of the corporate sample*

**Table 9: Industries(Vedran Cengic 2016)** & **Table 10: Countries((Vedran Cengic 2016)**

| INDUSTRY | TOTAL |
|---|---|
| INTERNET & COMPUTER SERVICES:(SOFTWARE/ HARDWARE) | 584 |
| FOOD & WHOLESALE | 290 |
| REAL ESTATE | 285 |
| INDUSTRIAL MACHINERY & SUPPLIES | 284 |
| MEDIA | 272 |
| ENERGY | 232 |
| MEDICAL EQUIPMENT & PHARMA | 200 |
| CLOTHING & ACCESSORY | 195 |
| ELECTRONICS | 189 |
| AUTO INDUSTRY | 171 |
| HOUSEHOLD & PERSONAL PRODUCTS | 147 |
| TELECOM | 146 |
| BUILDING MAT.& FIX. | 129 |
| CHEMICALS | 125 |
| BUSINESS SUPPORT SVS. | 124 |
| HEAVY CONSTRUCTION | 113 |
| BROADLINE & SPECIALITY RETAILERS | 105 |
| RECREATION SERVICES | 86 |
| BIOTECHNOLOGY | 84 |
| HOTELS, RESTAURANTS & BARS | 79 |
| OFFICE SUPPLIES | 78 |
| BREWERS & VINTNERS | 77 |
| AIRLINE & AEROSPACE | 39 |
| OTHER | 1327 |

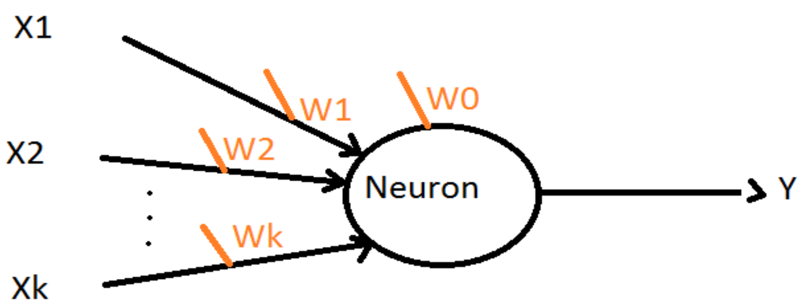
| COUNTRY | TOTAL (%) |
|---|---|
| FRANCE | 29.0 |
| GERMANY | 27.0 |
| ITALY | 7.0 |
| GREECE | 6.7 |
| NETHERLANDS | 4.7 |
| SPAIN | 4.5 |
| BELGIUM | 4.2 |
| FINLAND | 3.8 |
| AUSTRIA | 2.7 |
| CYPRUS | 2.0 |
| PORTUGAL | 1.6 |
| IRELAND | 1.1 |
| SLOVENIA | 1.0 |
| LITHUANIA | 0.9 |
| LUXEMBOURG | 0.85 |
| LATVIA | 0.7 |
| SWITZERLAND | 0.6 |
| OTHER EU | 1.65 |

## ***Appendix 3: Neural networking***

### 3.1: Single neuron

This short introduction will focus only on the case of a single neuron. Although 10 neurons are used, the essence of the problem remains the same. A neuron is essentially nothing more than some mathematical function mapping many input variables to only one response $f: X \rightarrow Y$..

**Figure 14: Illustration of single neuron (Vedran Cengic 2016)**




A single neuron has a number of inputs, parameters, and one output. The parameters are called weights. What the neuron does is: it adds up the weighted sum of its inputs using its own weights and adds one more number called $W_0$, this is called the bias in what is called the activation function ' a '. Figure 14 illustrates the case of a single neuron.

$$a = \sum_{k=1}^{K} W_k X_k + W_0$$

The output Y is than computed by some function with the activation as input: ***Y***= *f(a).* That function could be a logistic function, an hyperbolic tangent, or a step function

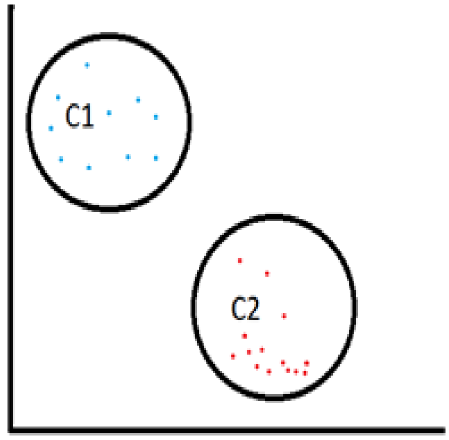



**Figure 15: Classification of the data in 2 distinct groups (Vedran Cengic 2016)**

***(If you have a data point x and you are wondering what the probability is that x belongs to the group C1 it would be: p(c1|x)=f(a). With the activation as discussed above.)***

### 3.2: Learning

Having specified the activation function, one still has to find a way to determine the optimal weights. This process is also called 'learning' (nothing more than adjusting the weights). Suppose you have a sample of data points X and known outputs or targets $T: \{X^{(n)}, T^{(n)}\}$. What learning does, is adjusting the weights (W) of the parameters X so that the output $Y(X^{(n)},\ W)$ is as close as possible to the targets T. In this section the process for only 2 distinct targets will be illustrated. The process for more than 2 targets, as it is the case in this thesis with 7 distinct targets (the 7 different rating classes), is very similar. First an objective function $G(W)$ to minimize as a function of the weights is constructed. In the case of only 2 targets it could be the following function:

$$G(W) = -\sum_{i}^{N} t_n * \ log\, y_n\left(x^{(n)}; W\right) + (1 - t_n) * log\ (1 - y_n(x^{(n)}; W))$$

Using this function will result in Figure 16 below, where the output Y is projected on the horizontal axis and the 2 targets ( 0 or 1 ) are mapped in function of the output Y. The closer Y is to 1 the higher the probability that it will be classified as target 0 and vice versa

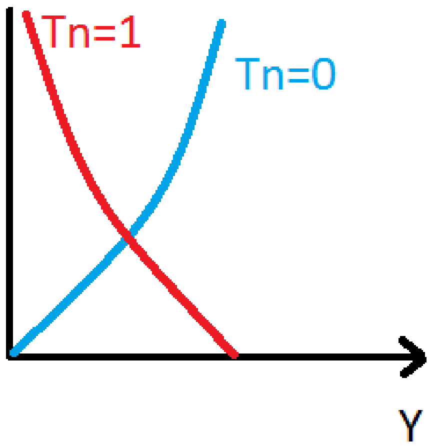



**Figure 16: Classification with only 2 targets (Vedran Cengic 2016)**

More generally if a multivariable function F(X) is differentiable in a neighborhood ' a ', then F(X) decreases fastest if one goes from a in the direction of the negative gradient. It can be showed that the gradient ( $g$) is computed as follows:

$$g = \sum_{n} (t_n - y_n) * x^{(n)}$$

Then the weights are changed according to the gradient descent method with the parameter η as the learning rate. This parameter should be specified ad hoc.

$$\Delta W = -\eta\, g$$

This method however can generate big weights as the optimal solution which is not preferable. In order to avoid big weights de objective function *G(W)* is adjusted with a penalty for big weights also known as learning with regularization.

$$G(W) = -\sum_{i}^{N} t_n * \ log\, y_n\left(x^{(n)}; W\right) + (1 - t_n) * log\ (1 - y_n(x^{(n)}; W)) +\ \alpha * E(W)$$

with

$$E(W) = \frac{1}{2}\sum_{k=1}^{K} W_k$$

$\alpha$ is called the 'regularization constant' and E(W) the regularizer. Alpha is again a constant which needs to be set ad hoc. There are methods to determine the $\alpha$, however it does not fit in the scope of this paper.

### 3.3 Pattern recognition

For the purpose of this thesis a family of neural networking is used called pattern recognition. The theory remains however very similar. First the transition matrices are computed without filling the Single and Double gaps. Second, after obtaining the deviations and regressing to the macroeconomic variables a selection of preferable variables is made. Finally, these variables will be the input or X variables discussed in the 3.2 along with the current credit rating in year t and the credit rating in the previous year t-1. For the targets T the ratings of the corporates considered in year t+1 is used. The process is very similar to what was discussed in the previous section with one major difference: instead of having two possible targets and working in $R^2$, now there are 7 targets corresponding to the 7 credit ratings and computations are done in $R^7$. In order to be computable the credit ratings in the inputs are mapped to their numerical values : {AAA, AA ,A, BBB, BB, B, CCC, CC C}-> {1, 2, 3 , 4, 5, 6, 7}. The output however is a 7x1 vector containing values between {0,1}. Therefore in order to be able to classify the input variables to a target rating, the target ratings have to be mapped to an 7x7 matrix:

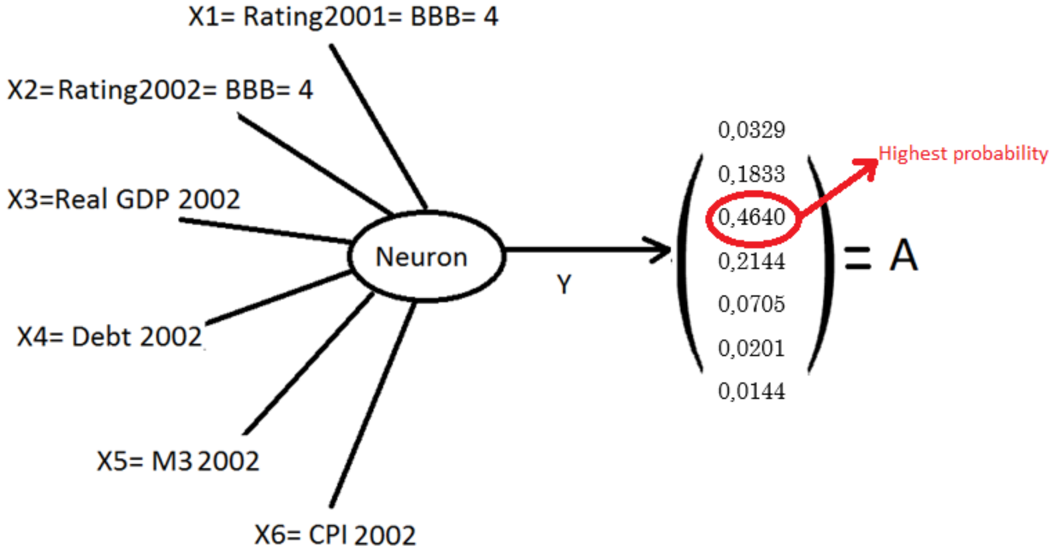
$$\begin{pmatrix} \boldsymbol{AAA} & \boldsymbol{AA} & \boldsymbol{A} & \boldsymbol{BBB} & \boldsymbol{BB} & \boldsymbol{B} & \boldsymbol{CCC'CC'C} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

This is similar to the example in Figure 16 where there are only two targets in $R^2$)

If for example  company Z is missing data in a particular year, the estimation of the weights is done using the ratings of all the companies in the sample as targets, evidently those who are not missing, and their ratings the previous two years as well as the macroeconomic variables selected by the MF- model. When the neurons are done learning and an optimal set of weights is determined, the weights are then applied to the input variables of company Z. As a result the most likely rating will be obtained for company Z in that year, given patterns already observed in the sample and the macroeconomic environment. This training is done over a 1000 times and each time the weights are extracted and stored. After 1000 iterations the weighted average of the weights is taken.

This process will be illustrated in Figure 17 using a simplified example. Suppose company Z is only missing data in 2002 and the rating in 2001 and 2000 were BBB an BBB respectively. Furthermore suppose a set of 4 macroeconomic variables are selected: The real GDP, the CPI index, Debt and Money Supply (M3), still using a single neuron a possible output could be:

**Figure 17: Illustration of pattern recognition (Vedran Cengic 2016)**




According to this model, an upwards pattern is observed in the year 2002 and given the economic conditions that year and that the rating was BBB the previous two years, the most likely credit migration for company Z will be A with a probability of 46.4%.

## ***Appendix 4: Example of S&P transition matrix***

**Figure 18: Example of S&P transition matrix (Banque du France 2013)**

| | AAA | AA | A | BBB | BB | B | CCC,CC,C | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 90,2% | 8,9% | 0,5% | 0,2% | 0,0% | 0,0% | 0,2% | 0,0% |
| AA | 0,3% | 89,3% | 9,8% | 0,6% | 0,0% | 0,0% | 0,0% | 0,0% |
| A | 0,0% | 2,1% | 92,3% | 5,1% | 0,2% | 0,1% | 0,0% | 0,0% |
| BBB | 0,0% | 0,1% | 4,2% | 91,2% | 3,4% | 0,7% | 0,1% | 0,2% |
| BB | 0,0% | 0,1% | 0,4% | 5,1% | 86,9% | 6,4% | 0,5% | 0,6% |
| B | 0,0% | 0,0% | 0,2% | 0,5% | 7,1% | 82,7% | 4,8% | 4,7% |
| CCC,CC,C | 0,0% | 0,0% | 0,3% | 0,6% | 1,1% | 13,0% | 58,1% | 27,1% |
| D | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 0,0% | 100,0% |

Table 1: S&P average credit rating transition matrix (1990-2011)

## ***Appendix 5: Estimates of the rating group 2 and 3***

**Table 11: Model 1 MEDIUM rating group (Vedran Cengic 2016)**

| | CORRELATION | | | ESTIMATES | P-VALUE | ADJ. $R^2$ |
|---|---|---|---|---|---|---|
| CONSTANT | 1 | -0.97 | 0.093 | -0.01 | $2.16X10^{-5}$ | **0.725** |
| G. BOND YIELD | -0.968 | 1 | -0.16 | 0.002 | $1.92X10^{-5}$ | |
| SALES | 0.093 | -0.16 | 1 | 0.061 | 0.01579 | |

**Table 12: Model 2 MEDIUM rating group (Vedran Cengic 2016)**

| | CORRELATION | | | ESTIMATES | P-VALUE | ADJ. $R^2$ |
|---|---|---|---|---|---|---|
| CONSTANT | 1 | -0.96 | -0.05 | -0.01 | $2.28X10^{-5}$ | **0.686** |
| G. BOND YIELD | -0.958 | 1 | -0.11 | 0.002 | $2.64X10^{-5}$ | |
| GDP DEF | -0.045 | -0.11 | 1 | 0.005 | 0.0523 | |

**Table 13: Model 1 BAD rating group (Vedran Cengic 2016)**

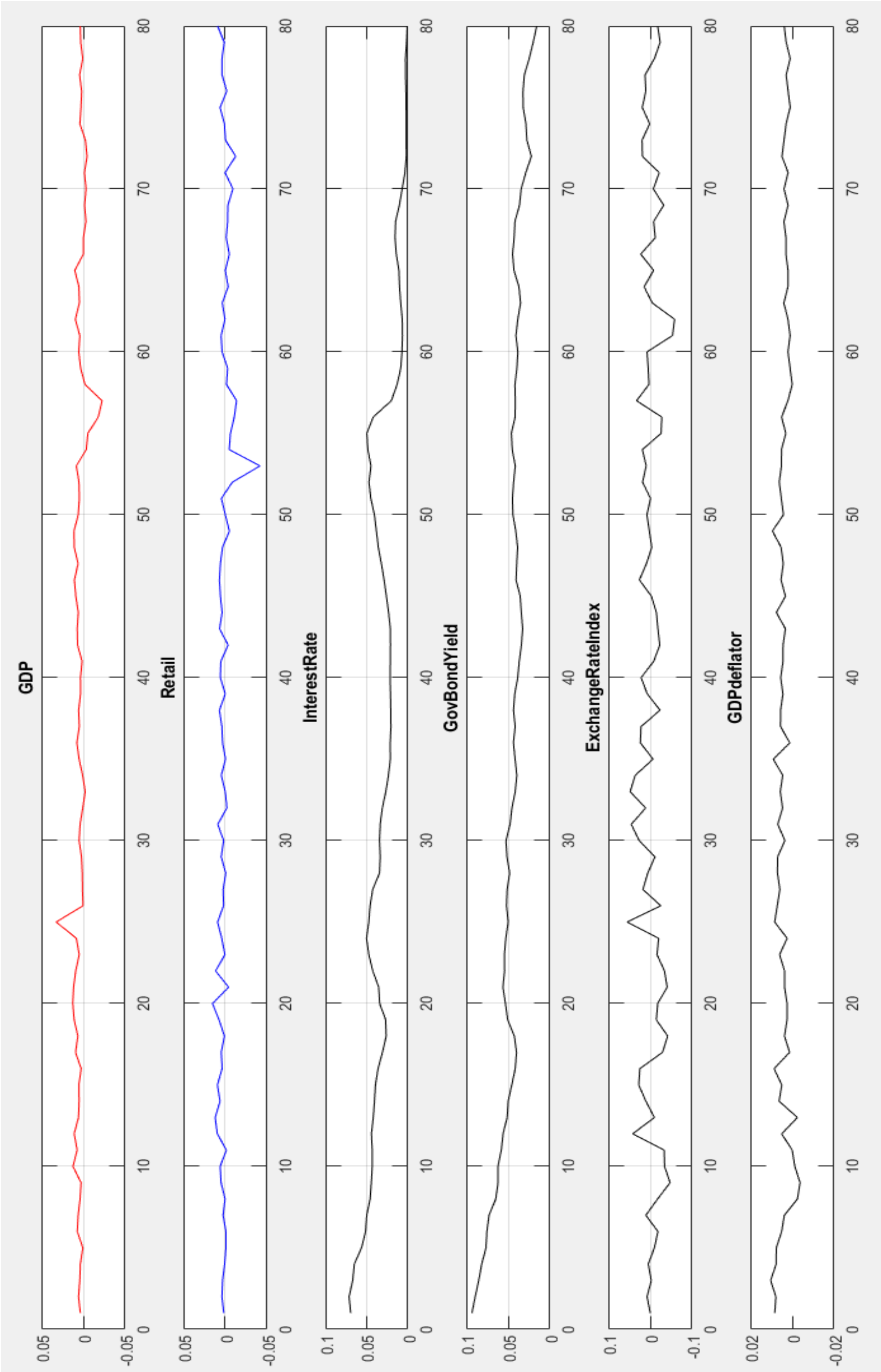
| | CORRELATION | | | ESTIMATES | P-VALUE | ADJ. $R^2$ |
|---|---|---|---|---|---|---|
| CONSTANT | 1 | -0.97 | 0.093 | -0.011 | $1.47X10^{-5}$ | **0.747** |
| G. BOND YIELD | -0.968 | 1 | -0.16 | 0.0025 | $1.11X10^{-5}$ | |
| SALES | 0.093 | -0.16 | 1 | 0.0665 | 0.0097 | |

**Table 14: Model 2 BAD rating group (Vedran Cengic 2016)**

| | CORRELATION | | | ESTIMATES | P-VALUE | ADJ. $R^2$ |
|---|---|---|---|---|---|---|
| CONSTANT | 1 | -0.96 | -0.05 | -0.0116 | $3.23X10^{-5}$ | **0.672** |
| G. BOND YIELD | -0.958 | 1 | -0.11 | 0.0026 | $2.86X10^{-5}$ | |
| GDP DEF | -0.045 | -0.11 | 1 | 0.0443 | 0.1052 | |

## ***Appendix 6: Quarterly evolution of the VAR variables from 1995-2014***

**Figure 19: Stationary evolution of VAR variables: 80 Quarters (1995-2014) (Vedran Cengic 2016)**



## *Appendix 7: Estimates of the VAR (4) model*

**Figure 20: Estimates of the VAR(4) model (Vedran Cengic 2016)**

```
  Model    : 6-D VAR(4) with Additive Constant
  Info     : Macro stress test
             Conditional mean is AR-stable and is MA-invertible
  Series : GDP
  Series : retail
  Series : InterestRate
  Series : Yield
  Series : ExchangeRate
  Series : Deflator

a Constant:
      0.00747648
      0.00125428
      0.00285001
       0.0035139
      0.00940329
      0.00448788

AR(1) Autoregression Matrix:
        0.106036        0.113038         0.111679      -0.0348466      -0.0519847       -0.511993
       0.0379533        0.153305        -0.528322        0.449785     -0.00595846       -0.266751
        0.187526       -0.120193          1.00686       0.0106955      -0.0214667       -0.167456
        0.140916      -0.0417208        -0.280272         1.12709      -0.0184796       0.0136802
       -0.577858       -0.482915          1.34784       -0.974237         0.35739        0.168836
       0.0398225       0.0126561         0.199734       -0.129533     -0.00137703        0.108324

AR(2) Autoregression Matrix:
       -0.110615        0.177171        0.0955151       -0.116497     -0.00959299      -0.0263308
        0.181219        0.127202         0.522973       -0.394108      -0.0151563     0.000594307
        0.128644      -0.0817822        -0.111118       0.0201052     -0.00190437       -0.212601
        0.112193       -0.058883         0.331528       -0.242892      0.00789245      -0.0755872
        0.198283         1.02615         -3.25333         2.00691       -0.113348         2.18395
      -0.0620133       0.0536817        -0.211107        0.152855       0.0080811        0.308913

AR(3) Autoregression Matrix:
       -0.157028         0.43218        -0.188004       0.0884613      -0.0447262        0.380618
      -0.0491565        0.217166        -0.105864      -0.0136548     -0.00747127        0.309328
       0.0454354       0.0719757       -0.0444286       0.0566902      -0.0215229       0.0137916
    -0.00905265       0.0457766       -0.0790912      0.00963262      -0.0357418        0.327521
       -0.826295        0.695528          3.64272          -2.431       0.0807672        -1.14719
      -0.0110937       0.0290241         0.149171       -0.121598      0.00114524      -0.0582204

AR(4) Autoregression Matrix:
       -0.234236        0.327475         0.111705      -0.0946457      -0.0452302        0.171857
       -0.244813        0.231142        0.0814664       0.0280831       0.0241109       -0.788997
      -0.0622611        0.286751        0.0912162       -0.164404      -0.0146327        0.368722
      0.00351484      0.00363566        0.0793599      -0.0727337    -0.000692631       0.0195641
        0.572606        -1.05055          -1.4444         0.88596        -0.22156        0.348411
       0.0432635      0.00412072       -0.0776222       0.0150479       0.0161084     -0.00690637

Q Innovations Covariance:
     1.73162e-05     2.28771e-06      2.67736e-06     1.90454e-06      8.8203e-06    -6.42555e-08
     2.28771e-06     3.14372e-05      3.11456e-06     6.20647e-07     1.44018e-05    -2.61014e-07
     2.67736e-06     3.11456e-06      3.61012e-06     6.48511e-07     9.55112e-06     3.29763e-08
     1.90454e-06     6.20647e-07      6.48511e-07     5.61546e-06    -9.69196e-07     1.13933e-07
      8.8203e-06     1.44018e-05      9.55112e-06    -9.69196e-07     0.000319282     1.23361e-05
    -6.42555e-08    -2.61014e-07      3.29763e-08     1.13933e-07     1.23361e-05     3.43841e-06
```

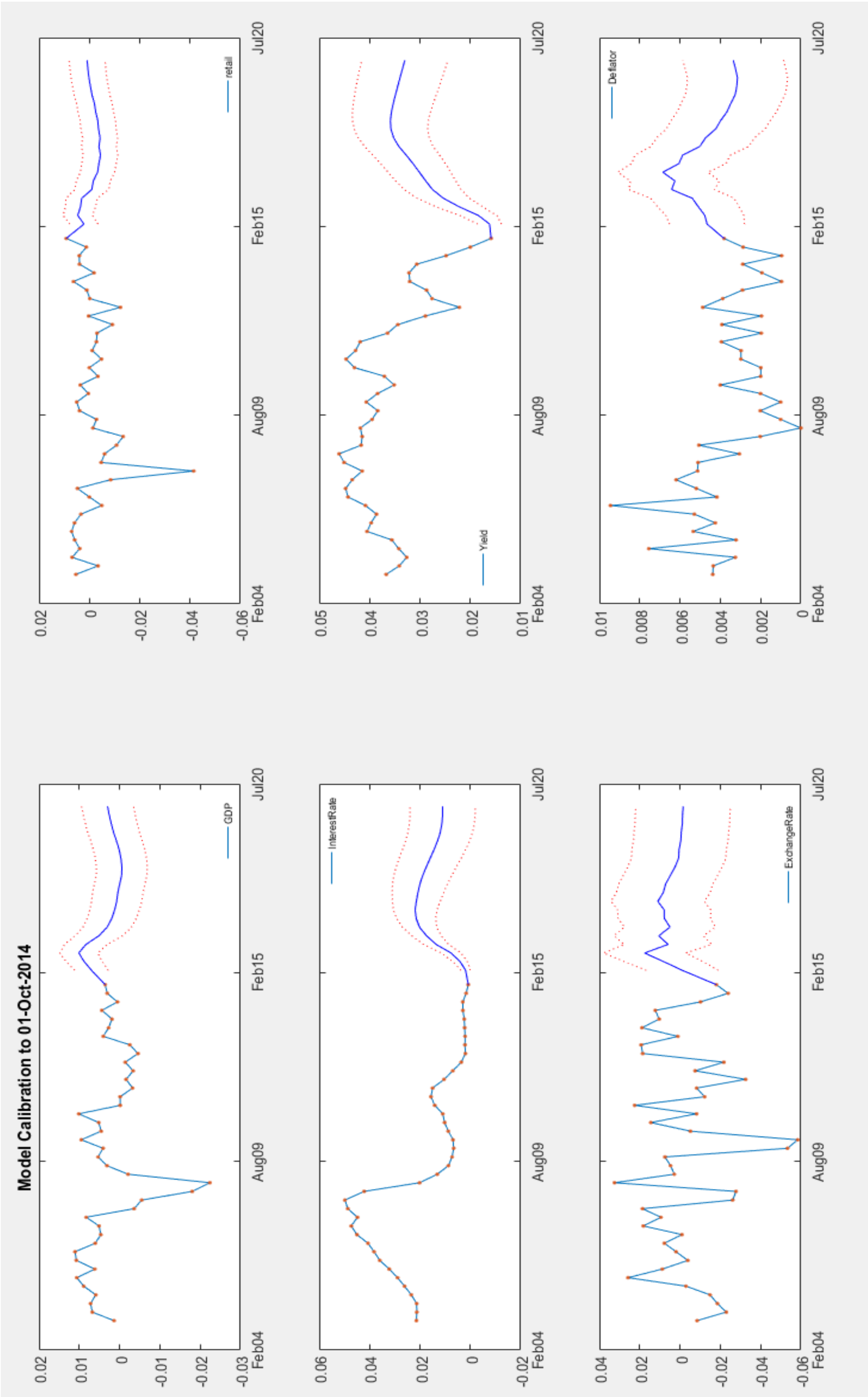
## *Appendix 8: Forecasting errors*

**Table 15: Percentage RMSE from actual values (Vedran Cengic 2016)**

| VARIABLE | GDP | SALES | IR RATES | YIELD | EX. RATES | GDP DEF |
|---|---|---|---|---|---|---|
| 2011 | 0.62 | 0.83 | 4.19 | 5.15 | 4.42 | 0.29 |
| 2012 | 0.58 | 0.99 | 2.06 | 8.07 | 6.83 | 0.11 |
| 2013 | 1.19 | 2.65 | 0.65 | 2.86 | 1.48 | 0.16 |
| 2014 | 0.13 | 1.03 | 1.84 | 7.00 | 1.51 | 0.46 |

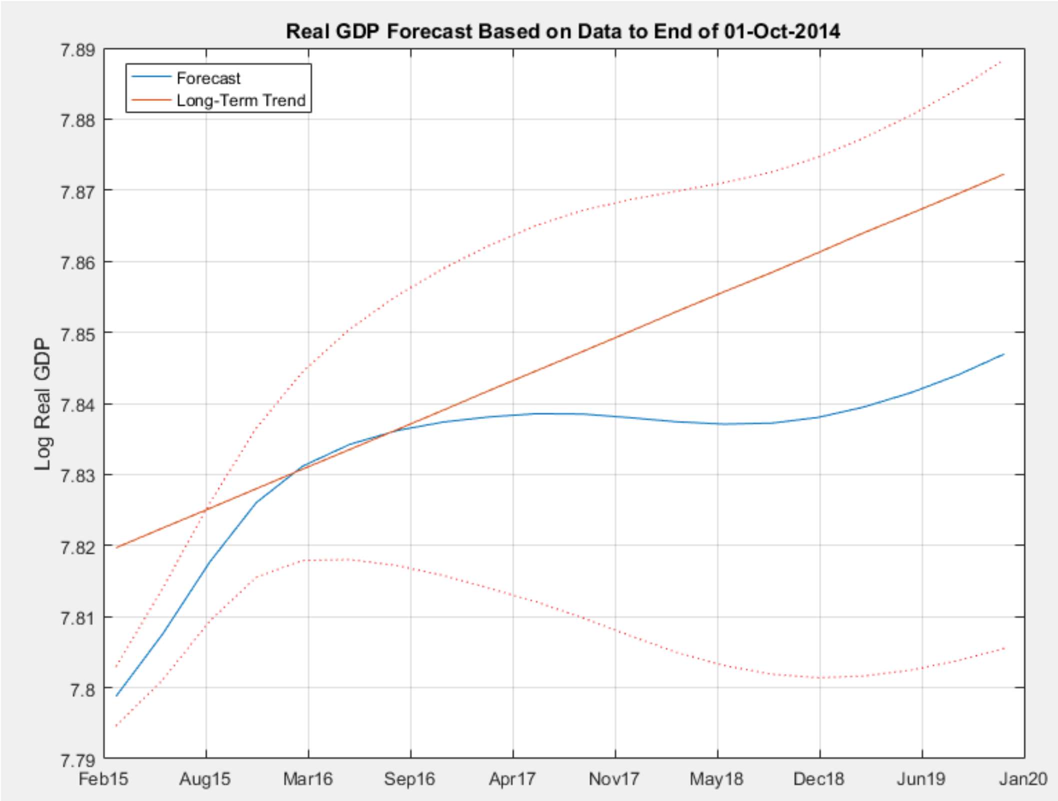
## ***Appendix 9: Forecasts/ Baseline***

**Figure 21: Baseline Forecasts  (Vedran Cengic 2016)**



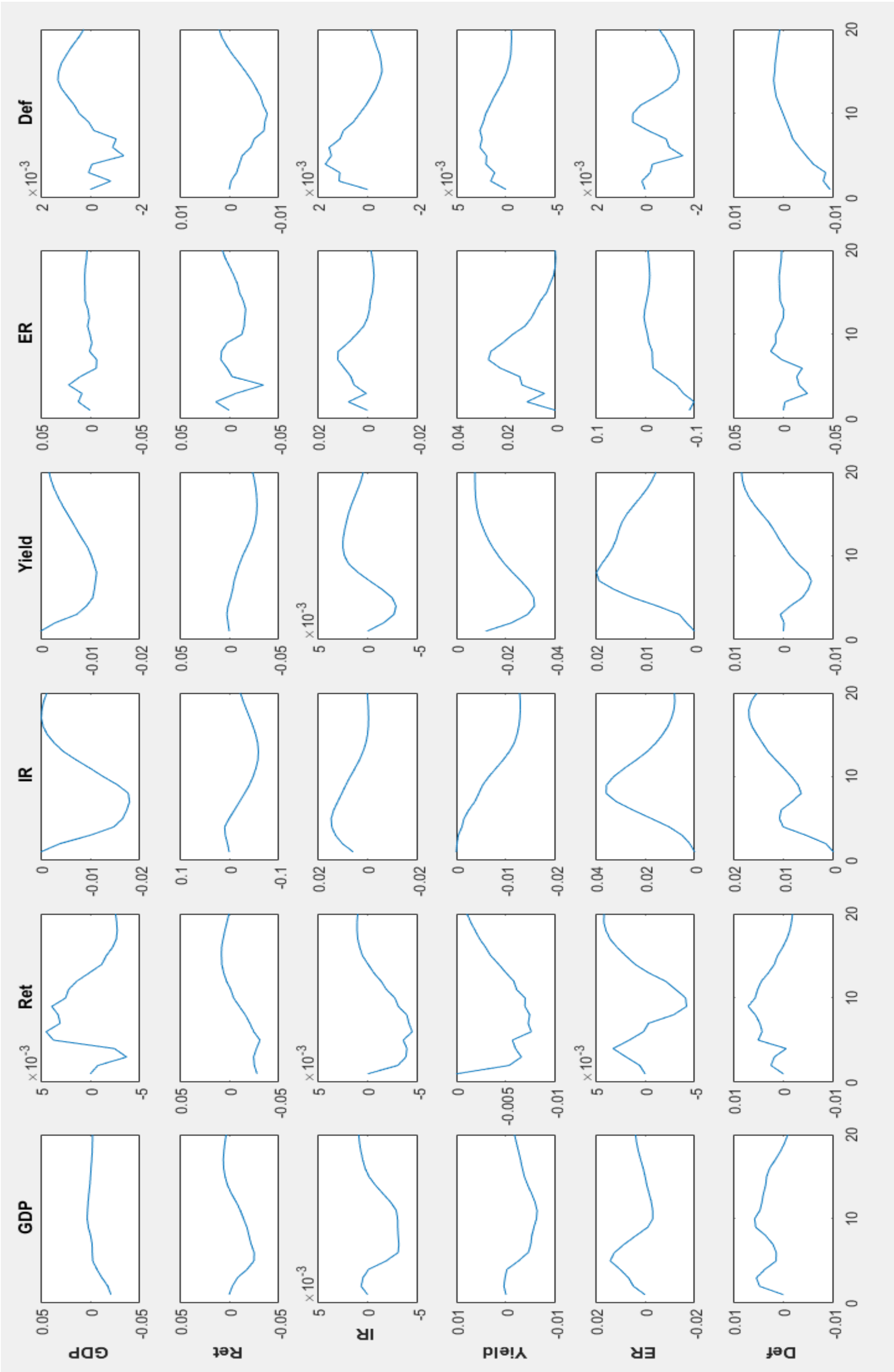
## ***Appendix 10: Real GDP forecast***

**Figure 22: Baseline Forecast of the real GDP (Vedran Cengic 2016)**



## ***Appendix 11: Impulse-responses***

**Figure 23: Responses to shocks compared to the baseline (Vedran Cengic 2016)**



## ***Appendix 12: Implied transition matrices Baseline Scenario***

TRANSITION_MATRIX_BASELINE_SCENARIO_2015 =

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.8007 | 0.0969 | 0.0311 | 0.0150 | 0.0059 | 0.0023 | 0.0482 |
| 0.0996 | 0.6975 | 0.1043 | 0.0369 | 0.0107 | 0.0038 | 0.0472 |
| 0.0360 | 0.1077 | 0.6462 | 0.1111 | 0.0352 | 0.0128 | 0.0509 |
| 0.0205 | 0.0470 | 0.1079 | 0.6470 | 0.0813 | 0.0327 | 0.0636 |
| 0.0141 | 0.0233 | 0.0519 | 0.1174 | 0.6231 | 0.0749 | 0.0953 |
| 0.0121 | 0.0161 | 0.0325 | 0.0677 | 0.0961 | 0.6247 | 0.1508 |
| 0.0167 | 0.0129 | 0.0178 | 0.0362 | 0.0410 | 0.0501 | 0.8253 |

TRANSITION_MATRIX_BASELINE_SCENARIO_2016 =

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.8096 | 0.0997 | 0.0322 | 0.0156 | 0.0062 | 0.0024 | 0.0345 |
| 0.0997 | 0.7063 | 0.1073 | 0.0383 | 0.0111 | 0.0040 | 0.0334 |
| 0.0361 | 0.1087 | 0.6585 | 0.1112 | 0.0343 | 0.0122 | 0.0390 |
| 0.0206 | 0.0473 | 0.1090 | 0.6592 | 0.0810 | 0.0319 | 0.0511 |
| 0.0141 | 0.0234 | 0.0522 | 0.1189 | 0.6348 | 0.0746 | 0.0820 |
| 0.0121 | 0.0161 | 0.0328 | 0.0685 | 0.0979 | 0.6386 | 0.1340 |
| 0.0168 | 0.0130 | 0.0179 | 0.0365 | 0.0415 | 0.0510 | 0.8234 |

TRANSITION_MATRIX_BASELINE_SCENARIO_2017 =

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.8137 | 0.1010 | 0.0327 | 0.0159 | 0.0063 | 0.0024 | 0.0280 |
| 0.0998 | 0.7103 | 0.1087 | 0.0389 | 0.0114 | 0.0041 | 0.0269 |
| 0.0362 | 0.1089 | 0.6611 | 0.1112 | 0.0341 | 0.0121 | 0.0365 |
| 0.0206 | 0.0473 | 0.1093 | 0.6618 | 0.0809 | 0.0317 | 0.0484 |
| 0.0142 | 0.0234 | 0.0523 | 0.1192 | 0.6373 | 0.0745 | 0.0791 |
| 0.0122 | 0.0161 | 0.0328 | 0.0687 | 0.0982 | 0.6413 | 0.1307 |
| 0.0168 | 0.0130 | 0.0179 | 0.0366 | 0.0416 | 0.0511 | 0.8230 |

## ***Appendix 13: Implied transition matrices Adverse Scenario***

TRANSITION_MATRIX_ADVERSE_SCENARIO_2015 =

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.7942 | 0.0950 | 0.0303 | 0.0146 | 0.0057 | 0.0022 | 0.0580 |
| 0.0995 | 0.6912 | 0.1022 | 0.0360 | 0.0104 | 0.0037 | 0.0570 |
| 0.0359 | 0.1066 | 0.6320 | 0.1107 | 0.0359 | 0.0132 | 0.0655 |
| 0.0205 | 0.0467 | 0.1065 | 0.6330 | 0.0812 | 0.0334 | 0.0787 |
| 0.0141 | 0.0232 | 0.0514 | 0.1158 | 0.6095 | 0.0749 | 0.1111 |
| 0.0121 | 0.0160 | 0.0322 | 0.0667 | 0.0941 | 0.6077 | 0.1711 |
| 0.0167 | 0.0128 | 0.0176 | 0.0358 | 0.0403 | 0.0492 | 0.8276 |

TRANSITION_MATRIX_ADVERSE_SCENARIO_2016 =

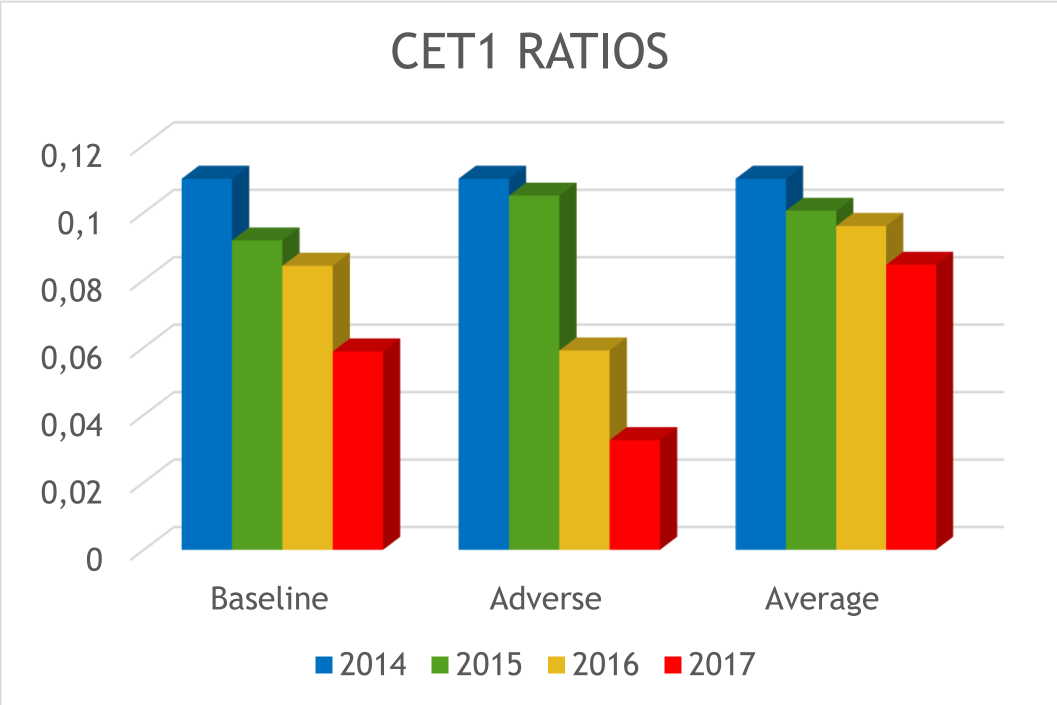
| | | | | | | |
|---|---|---|---|---|---|---|
| 0.7966 | 0.0957 | 0.0306 | 0.0147 | 0.0058 | 0.0022 | 0.0543 |
| 0.0996 | 0.6936 | 0.1030 | 0.0363 | 0.0105 | 0.0037 | 0.0533 |
| 0.0360 | 0.1068 | 0.6343 | 0.1108 | 0.0358 | 0.0132 | 0.0631 |
| 0.0205 | 0.0468 | 0.1067 | 0.6352 | 0.0812 | 0.0333 | 0.0762 |
| 0.0141 | 0.0232 | 0.0515 | 0.1160 | 0.6117 | 0.0750 | 0.1085 |
| 0.0121 | 0.0160 | 0.0323 | 0.0669 | 0.0944 | 0.6102 | 0.1681 |
| 0.0167 | 0.0129 | 0.0177 | 0.0358 | 0.0404 | 0.0493 | 0.8272 |

TRANSITION_MATRIX_ADVERSE_SCENARIO_2017 =

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.8031 | 0.0977 | 0.0314 | 0.0151 | 0.0060 | 0.0023 | 0.0445 |
| 0.0996 | 0.6999 | 0.1051 | 0.0373 | 0.0108 | 0.0038 | 0.0434 |
| 0.0360 | 0.1076 | 0.6443 | 0.1111 | 0.0353 | 0.0128 | 0.0529 |
| 0.0205 | 0.0470 | 0.1077 | 0.6451 | 0.0813 | 0.0328 | 0.0656 |
| 0.0141 | 0.0233 | 0.0518 | 0.1172 | 0.6213 | 0.0749 | 0.0974 |
| 0.0121 | 0.0160 | 0.0325 | 0.0675 | 0.0958 | 0.6219 | 0.1542 |
| 0.0167 | 0.0129 | 0.0177 | 0.0361 | 0.0408 | 0.0500 | 0.8257 |

## ***Appendix 14: CET1 ratios***

**Figure 24: Overview of the CET1 ratios in different scenarios (Vedran Cengic 2016)**



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