FACULTY OF BUSINESS AND ECONOMICS
DEPARTEMENT OF ACCOUNTANCY, FINANCE AND INSURANCE

KATHOLIEKE
UNIVERSITEIT
LEUVEN

On the use of risk-free rates in the discounting of insurance cash flows

Thesis submitted to obtain the degree of
Master of Science in Financial and Actuarial Engineering

Ir. Gil Delcour

Promotors:
Prof. dr. Jan Dhaene
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Acknowledgments

This master paper would never have been what it has become today without the help and support of the following people. Thank you:

Professor dr. Jan Dhaene, and Professor dr. Schoutens, for making my interest in life insurances and finance models grow, for being my promotors and for trusting me and giving me the liberty to do it my way.

Nathalie Vanderbecken, for your understanding when I needed to study or work on this master paper (even if that meant more work for you), for the interesting discussions on everything related market-consistency, and for carefully reading a draft version of this master paper.

Erik Nelis, for being my Solvency II newsfeed, who always knows where to find back a document, and for carefully reading a draft version of this master paper.

Sandra Toscano, for helping me find my way in Bloomberg.

Dr. ir. Korneel van den Broek, although we didn't always agree, for the interesting discussions on liquidity premiums and matching premiums.

My parents, for giving me the opportunity to finish this third master.

Jan and Martine, for always wanting the best for me.

Lili, my fiancée, for your never-ending support and love.

Gil Delcour

ABSTRACT This master paper discusses which interest rate should be used in a market-consistent valuation model of insurance liabilities. Such models are required for Market Consistent Embedded Value (MCEV) reporting, and for Solvency II purposes.

For use as a basic risk-free rate, we show that swap rates are probably better choices than government bond rates and interbank rates. Swap rates tend to have lower credit risk, and are available for high maturities, the latter being particularly needed for use in insurance liability valuation models. Swap rates, adjusted for credit spread, are indeed the rates currently prescribed as the basic risk-free interest rates to use in both MCEV and Solvency II. Nevertheless, we show that there is considerable evidence indicating that the current credit spread in swap rates could be up to ten times higher then the currently estimated value of 10bp.

Insurers feel that because of the illiquidity of their liabilities, and because they feel they are not exposed to certain risks, they should be allowed the use of risk adjustments in the discount rate used for the discounting of their liabilities. Such derogations are allowed under the MCEV Principles, and under the Solvency II QIS5 Technical Specifications in the form of a liquidity premium. In the Draft Implementing Measures Solvency II, the application of a matching premium or a counter-cyclical premium is allowed, but the requirements have become much more restrictive.

The use of these premiums has always been controversial, as some believes it leads to non-market-consistency. However, we show that by changing a questionable assumption made in a paper opposing the use of such premiums [1] that the use of a liquidity premium can lead to a market-consistent approach. Because the example in the paper is based on quite some restrictions, we are not yet able to make conclusions about whether the use of a liquidity premium is justified in more general cases as well. One of these restrictions is the absence of an early surrender option. A preliminary analysis made indicates that when a surrender option is included, the use of a liquidity premium might be invalid.

Because the matching premium is much more restrictive in its applicability, and because the restrictions show a lot of similarity with the restrictions under which we showed the use of a liquidity premium might be justified, we believe the matching premium might be justified as well. However, we see two key issues. Firstly, it is not clear whether these restrictions are the largest set under which a premium is justified. Secondly, the estimation of the magnitude of the matching premium is currently based on an estimation method for that part of the spread which does not correspond to an average default probability over a long time period. It is not clear whether this full “non-fundamental spread” as it is called, should be added to the risk-free rate, even under the strict restrictions of the matching premium. Furthermore, the use of an average default probability over time is felt to be a very bad, and unprudent approximation for the current default probability.

Because of the possible pro-cyclical consequences of using a market-consistent approach, we believe in the necessity of counter-cyclical measures. Nevertheless, we believe that the use of a counter-cyclical premium, which is an adaptation of the risk-free rate as counter-cyclical measure, is a bad idea. Although we believe that a counter-cyclical premium might indeed have a counter-cyclical effect, we feel that if there is no theoretical justification for applying a premium, that no premium should be applied. If it were to be applied without a theoretical justification, this would lead to meaningless and intransparant numbers, on which no sound risk managment can be based. Therefore, other counter-cyclical measures than a counter-cyclical premium should be looked for.

Contents

List of Abbreviations

AMICE	Association of Mutual Insurers and Insurance Cooperatives in Europe
B+H	Barrie + Hibbert
bp	basis points (1bp = 0.01%)
CCP	Counter-Cyclical Premium
CDS	Credit Default Swap
CEA	Comité Européen des Assurances
CEIOPS	Committee of European Insurance and Occupational Pensions Supervisors
CF	Cash Flow
CRNHR	Cost of Residual Non Hedgeable Risk
DIM	Draft Implementing Measures (Solvency II)
EC	European Commission
ECB	European Central Bank
EEV	European Embedded Value
EIOPA	European Insurance and Occupational Pensions Authority
EONIA	Euro OverNight Indexed Average
Euribor	Euro Interbank Offered Rate
FV	Fair Value
GSP	Government Spread Premium
IE	Insurance Europe
IFRS	International Financial Reporting Standards
LIBOR	London Interbank Offered Rate
LP	(Il)liquidity Premium
MCEV	Market Consistent Embedded Value
MP	Matching Premium
NAV	Net Asset Value

OIS	Overnight Indexed Swap
OLO	Obligation Linaire/Lineaire Obligatie
OTC	Over-The-Counter
PS	Profit Sharing
PV	Present Value
QIS	Quantitative Impact Study
SST	Swiss Solvency Test
TEV	Traditional Embedded Value
TF	Task Force
TP	Technical Provisions
TTM	Time To Maturity
UFR	Ultimate Forward Rate
USD	United States Dollar
YTM	Yield To Maturity

List of Symbols

$\mathcal{A}_t$	Asset value at time t
α	Mean reversion parameter
C	Call option price
$\mathcal{C}_t$	Capital consumption at time t
e	Euler's constant (≈ 2.71828)
Γ_t	Binary default indicator at time t
K	Strike price
$\mathcal{L}_t$	Liability value at time t
p	Real-world probability
$P(t,T)$	Discount factor from time T to time t
$\mathbb{P}$	Real-world probability measure
π	Fair premium
q	Risk-neutral probability
$\mathbb{Q}$	Risk-neutral probability measure
r	(Risk-free) interest rate
s	Illiquidity spread
$S(t)$	Stock price
σ	Volatility
ς	Spread between corporate rate and risk-free rate
t	Time
T	Time at maturity
$V(t)$	Portfolio value

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Chapter 1

Introduction

Article 75 of the Framework Solvency II Directive[1] requires a fair value approach to the valuation of assets and liabilities [2]. This means that assets should be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction, while liabilities should be valued at the amount for which they could be transferred or settled, between knowledgeable willing parties in an arm's length transaction. Furthermore, Article 76 of the same directive explicitly requires the approach to be market-consistent.[2]

In practice, this more or less comes down to the fact that for assets, a mark-to-market approach should be taken, i.e. one uses market prices for the assets,[3] while for liabilities a mark-to-model approach should be used. When using a market-consistent approach, using a mark-to-model approach is assumed to be the best approximation one can obtain for an insurers liabilities portfolio, since trading of liabilities between different insurers does not happen that often, which means that market prices are either unavailable or at least not traded in a deep and liquid market.

The mark-to-model approach for liabilities gives insurers quite some freedom, which might be beneficial in the sense that insurers themselves tend to be experts in their own business and thus are well placed to make modelling decisions, but which also entails potential mispricing risks, as it is a current reality that there might be management rewards for higher calculated value and lower capital to be held. It is therefore very important that the principles and details are well agreed upon by both insurers and the regulators. The criteria to be used when deciding on these methodologies should give consideration to both theoretical and social arguments, as the main goal of Solvency II is to ensure better protection for the policyholders.

[1] Directive 2009/138/EC.

[2] For both articles, and for more information on Solvency II, see appendix B.

[3] It is assumed that there is a market and that market prices are known. This assumptions does not pose too much of a problem to the extent that the assets of the insurer are composed of bonds and equity, but could pose slight problems for mortgages, real estate and some other asset types. If no market is available for some assets, a mark-to-model approach for these assets is needed as well.

Although quite some aspects of the models to be used in Solvency II are not under too much discussion, a subject that certainly still is, is the determination of the risk-free interest rate that should be used to discount the stochastic liability cash flows. Insurers have found multiple ways and arguments to adjust the basic risk-free rate by using liquidity premia (LP), matching premia (MP) and counter-cyclical premia (CCP). All of these measures, including the basic risk-free rate to use, are discussed in this master paper, which is structured as follows:

Firstly, chapter 2 discusses some basic finance models, to make the reader familiar with no-arbitrage reasoning, and explain how these methods are now used in an insurance context, both in embedded value reporting and in Solvency II. It is shown why risk-free interest rates are of the utmost importance in a market-consistent approach, especially in an insurance context.[4] Furthermore, we discuss potential issues of using a market-consistent approach as the valuation method, most notably its potential pro-cyclical side effects.

Next, in chapter 3, different candidates that might be eligible for use as a basic risk-free rate are considered. A theoretical breakdown of real rates observed in the markets is put forward, which states that any rate consists of a risk-free rate plus some risk premiums. Different candidates for the risk-free rate are then discussed. Amongst government bonds, interbank rates and swap rates, swap rates classify the best as risk-free rate. Although swap rates are indeed currently agreed upon by the industry and the regulators to be used as basic risk-free rate in both embedded value and Solvency II, we raise an issue concerning the too low estimation of the credit spread which is currently part of the swap rates.

Then, chapter 4 and 5 deal with the LP, MP and CCP, which, as mentioned, are premiums insurers feel they should be allowed to add on top of their risk-free discount rate in the valuing of their liabilities. While the former chapter describes how these adjustments have been included in the past and will likely be included in the future in both embedded value reporting and Solvency II, the latter chapter tries to find out if these adjustments can be justified, or whether they are only smart actuarial finance ways to improve figures. By considering an example taken from a paper opposing the use of liquidity premia by Wüthrich [1], and by finding a flaw in the reasoning presented there, we show that under certain restrictions, one of these restrictions being the absence of a surrender option[5] the use of premiums can be justified and might possibly even be necessary. A preliminary study on the validity of using a premium when surrender options are present, indicates that a valid approach would need to be more sophisticated than an approach only using a premium in the discount rate.

Finally, chapter 6 gives an overview of the main outcomes, discovered issues, and urges for further research.

[4] Note that this master paper is written having in mind a life insurance company, and that when "insurance" is mentioned, mostly life insurance is meant.

[5] Or the presence of a surrender option, but with a perfect Market Value Adustment mechanism.

Chapter 2

Why are risk-free rates important?

Market-consistent valuation techniques have been used in the insurance industry for quite some years now. Since they will be necessary in Solvency II to calculate the value of the technical provisions, they will probably be used even more in the future. The core ideas wich are now used an insurance context, can be traced back to finance theory in the 1950s [3]. This chapter starts by discussing some basic finance models and introduces the notions of risk-neutral and real-world probability measures in section 2.1. The discussion is anything but comprehensive, since our goal is not to give an overview of finance pricing models, but to show why risk-free rates are important in these models, and since most of the discussed subjects are readily available in finance textbooks. In section 2.2, it is shown how and why this finance theory is now used in an insurance context. Even though market-consistent valuation or fair value can be questioned as a whole, see section 2.3, it is clear that in this framework the risk-free term structure is a vital parameter in insurance valuation, solvency and risk management in general, see section 2.4.

2.1 Some basic finance models

Suppose that you will receive 1000 in a year from now. A basic question is what that cash flow (CF) is worth today. If you make the assumption that you have a risk-free bank account,[1] which guarantees a yearly interest rate of 2%, and that you can equally borrow money from the bank at the same rate of 2%, then the answer is easy. Your CF of 1000 in a year from now is worth $\frac{1000}{1.02}$. If it would be worth less or more, than an *arbitrage opportunity* would arise, which means that you can gain a strictly positive amount of money without any initial investment.

Assume indeed that the value of the CF would be higher than $\frac{1000}{1.02}$. Then you would be able to lock in a strictly positive amount of money by selling a financial product which pays investors

[1] I.e. there is no doubt whether you will get back the money you put on there, and the interest rate is equally sure.

€1000 in a year from now, without the need of an initial investment. In the assumption that people would pay more for this than $\frac{1000}{1.02}$, say G, you put G on your risk-free bank account now, and in a year from now, your bank account will have grown to $1.02G > 1000$, leaving you with a strictly positive profit without an investment of your own.[2] Also, the price can't be lower than $\frac{1000}{1.02}$, since in that case you would borrow 1000 euro from the bank for a year, invest all of the money in the above product, paying a price per unit which is lower than $\frac{1000}{1.02}$. In the end, the financial product will pay you more than 1000 euro, which is the amount you have to pay back to the bank, again leaving you with a strictly positive profit without making an investment. Hence, this is an arbitrage opportunity.

This type of no-arbitrage reasoning is classic in finance. It is very often argued that arbitrage opportunities cannot exist, because if an arbitrage opportunity existed, everyone would exploit it, increasing or decreasing the price back to the no-arbitrage equilibrium. Arbitrage opportunities do exist however in some sectors,[3] but because of the argument just mentioned they are small and in any case they are not that easy to spot.

The example just presented considers the value of a *certain* future cash flow. The same type of reasoning can be used to value *uncertain* future cash flows, although modelling assumptions generally need to be made.[4]

Suppose that you would like to price a European call option, which has a payoff of $(S(T) - K)_+$ at time T, where $S(t)$ is e.g. some stock or index price at time t.

Let's start with a very basic model, known as the one step binomial tree model. Given the initial stock price $S(0) = S_0$, we suppose that the stock price at time T can be either S_0u, with a probability of $p = 87\%$, or S_0d, with a probability of $1 - p = 13\%$. Now consider an investor who wants to set up a strategy using a certain amount of stocks, and the call option. The total value of the portfolio can then be expressed as:

$$V(t) = S(t)\Delta - C(t) \tag{2.1}$$

where Δ is the amount of stock per negative amount of the call option in the portfolio. Denoting the payoff of the option by C_u when $S(T) = S_0u$, and by C_d when $S(T) = S_0d$, it is not hard to show that by taking

$$\Delta = \frac{C_d - C_u}{S_0u - S_0d} \tag{2.2}$$

[2] An alternative and equivalent strategy is to put $1000 < G$ on the bank account, leaving you with a positive profit at the time of selling the product.

[3] E.g. Domanski [4] mentions that traditionally, specialised financial traders in commodity markets focused on exploiting arbitrage opportunities. Typically, such opportunities arise as the consequence of commercial investors seeking to hedge their production or consumption in futures markets.

[4] Although interesting results can be obtained without modelling assumptions, see e.g. [5].

the payoff of the portfolio is equal in both states of the world. This means that an investor will be indifferent to owning this portfolio or owning a risk-free bank account which will have the same payoff at the same time. Therefore, he will pay the same price for these two investment strategies, which means that the present value of that portfolio should be equal to the payoff discounted at the risk-free rate, as shown above. In other words, the payoff of the portfolio can be replicated by a risk-free bank account. Doing the math,[5] we obtain that

$$C(0) = \mathrm{e}^{-rT}\left[qC_u + (1-q)C_d\right] \tag{2.3}$$

with

$$q = \frac{\mathrm{e}^{rT} - d}{u - d} \tag{2.4}$$

and where r is the continuous risk-free rate.[6] An alternative way to write this is to say that

$$C(0) = \mathrm{E}_{\mathbb{Q}}[\mathrm{e}^{-rT}C(T)] \tag{2.5}$$

or in words, the current price of the call option is the expected value of the discounted pay-off.

However, note that nowhere in the formulas, we made use of the probability that the stock would go up (p), neither of the probability that it would go down. Indeed equation 2.4 specifies probabilities q without using the real-world probabilities p. We call q the risk-neutral probability,[7] and $\mathbb{Q}$ the risk-neutral probability measure.

If we use $\mathbb{P}$ for the real-world probability measure, we can restate equation 2.5 as follows:

$$C(0) = \mathrm{E}_{\mathbb{P}}[D_T C(T)] \tag{2.6}$$

where D_T are called *deflators*. It can be proven that if a risk-neutral representation is possible, it is always possible to construct a real-world representation, by using the Girsanov theorem and the concept of Radon-Nykodim derivatives for changes of the probability measure [7, 8]. In this particular example it is not hard to see that $D_T = \mathrm{e}^{-rT}\frac{q}{p}$ in the up-state, and that $D_T = \mathrm{e}^{-rT}\frac{1-q}{1-p}$ in the down-state.

Although this real-world construction might seem a little artificial in the sense that it is presented here as derived from a risk-neutral analysis and twisted into using real-world probabilities by using a scaling factor, a real-world analysis is more appealing in the sense that it gives

[5] Or consulting e.g. the famous textbook by Hull [6].

[6] For a distinction between continuous and yearly rates, see appendix A.

[7] For a justification of the terminology, see e.g. [3].

an assessment of realistic risks, as they are based upon real-world scenarios.[8] The fact that they work with a real-world probability measure, also allows to introduce other real world probabilities, such as default probabilities, see e.g. Wüthrich [11, 1].

Of course the one-step binomial tree model just presented is very simplistic. The next step one could take is to introduce a multiple step binomial tree model. Again, it can be shown that equation 2.5 will continue to hold.[9]

The parameters u and d can then be chosen to be e.g. $\mathrm{e}^{\sigma\sqrt{\mathrm{d}t}}$ and $\mathrm{e}^{-\sigma\sqrt{\mathrm{d}t}}$,[10] where $\mathrm{d}t$ is the time step and σ is equal to the volatility[11] of the stock price process. If one then increases the number of steps and the step length thus becomes infinitesimally small, the binomial model converges to the famous Black-Scholes model [13] which was proven by Cox, Ross and Rubinstein in 1979 [12]. The main parameters in the Black-Scholes model are thus the volatility and the risk-free rate, which are chosen to be constant.

The conditions of the Black-Scholes model can be relaxed to obtain more general models. E.g. in the Heston model [14], a stochastic volatility process is introduced. More importantly for insurance,[12] is the introduction of a stochastic interest rate model.

There are two types of interest rate models, equilibrium models and no-arbitrage models [6]. The difference is that in an equilibrium model, today's term structure of interest rates is an output. One calibrates the parameters of the model to be consistent with the risk-free term structure. In a no-arbitrage model, today's term structure is an input of the model, and the parameters are calibrated to make the model consistent with observed, known, market prices. An example of a no-arbitrage model is the Hull-White model [15], which uses the following expression for the stochastic short rate ρ:[13]

$$\mathrm{d}\rho = (\theta(t) - \alpha\rho)\mathrm{d}t + \sigma\mathrm{d}W \tag{2.7}$$

where α is the mean-reversion rate, σ is the short rate volatility, $\mathrm{d}W \sim \mathrm{N}(0, \mathrm{d}t)$. And where

$$\theta(t) = r'(t) + \alpha r(t) + \frac{\sigma^2}{2\alpha}\left(1 - 2\mathrm{e}^{-2\alpha t}\right) \tag{2.8}$$

An advantage of the Hull-White model is that bond prices at time t in a Hull-White model can be expressed analytically, a disadvantage is that it allows negative interest rates.

[8] And although they are maybe harder to understand, they could potentially save on runtime [7]. Although computational capacity will probably still increase [9], barriers are beeing reached [10] and computation time is definitely an issue in practice.

[9] The formula for q has to be adjusted to the time step.

[10] These are the values proposed by Cox, Ross and Rubinstein [12, 8].

[11] It is not hard to prove that σ^2 then corresponds to the variance, at least if $\mathrm{d}t$ is small enough.

[12] Life insurers are particularly sensitive to interest rates and tend to invest largely in bonds.

[13] I.e. the continous rate, but in practice one often uses forward one year rates, see e.g. figure 2.1.

We shall not discuss different interest rate models and their pros and cons in the scope of this master paper, but it is important to note that in practice, mostly no-arbitrage models are used in the valuation of insurance liabilities, which means that the risk-free rate is a very important input to the model. The interest rate model can then be thought of as generating an interest rate probability distribution which is "around" the input deterministic risk-free term structure.[14]

In the examples presented above, closed form price solutions were available. For more complicated products or portfolios of products, and for more complicated models, this is often not the case. In this case, the solution can be estimated using Monte Carlo methods. This means that samples from the risk-neutral or real-world distribution are drawn, and the mean of the risk-free value in each of these scenarios is then used as a proxy of the risk-neutral expected value of the risk-free discounted cash flows.

In general, risk-neutral scenarios are Monte Carlo draws from the risk-neutral probability distribution of the underlying security prices on which the payouts of a derivative are dependent [16]. Applied to insurance and to interest rate models, the risk-neutral scenarios are interest rate scenarios which are Monte Carlo draws from the risk-neutral probability distribution of the interest rate model, on which the liabilities are dependent. Figure 2.1 presents seven interest rate scenarios of a Hull-White interest rate model and the risk-free term structure on which it is based.[15] As can be seen, the simulations presented are based on a discrete analogue of the continuous Hull-White model presented in equation 2.7, using time steps of 1 year. The rates presented are thus forward one year rates instead of continuous rates, for simulation simplicity.[16] In appendix D, figure D.1 shows the empirical percentiles of the risk-neutral distribution, obtained using 1000 risk-neutral simulations.

To conclude this section, it is important to note that things tend to get a little bit more complicated than this. By taking the shortcut of using directly a Monte Carlo approach and by using a risk-neutral scenario generator, it might be forgotten that the finance models presented above were based on no-arbitrage and replicating strategy arguments. Therefore, we have to mention the following. In general, it can be proven that if and only if their is no arbitrage, their exists an equivalent martingale [8],[17] and there is a unique replicating strategy for every

[14] This is illustrated in figure D.1.

[15] The data shown corresponds largely to the risk-free term structure used (incl. 75% liquidity premium, see further) at ageas, Axa and Allianz for year-end 2011 embedded value reporting [17, 18, 19]. The Hull-White model [15] has been calibrated on swaption market prices, using the risk-free term structure without liquidity premium as an input.

[16] For an explanation of the difference between forward one year rates and continuous rate, see appendix A.

[17] A martingale is a stochastic process in which the conditional expectation of the future value, given the history of the process, is the current value.

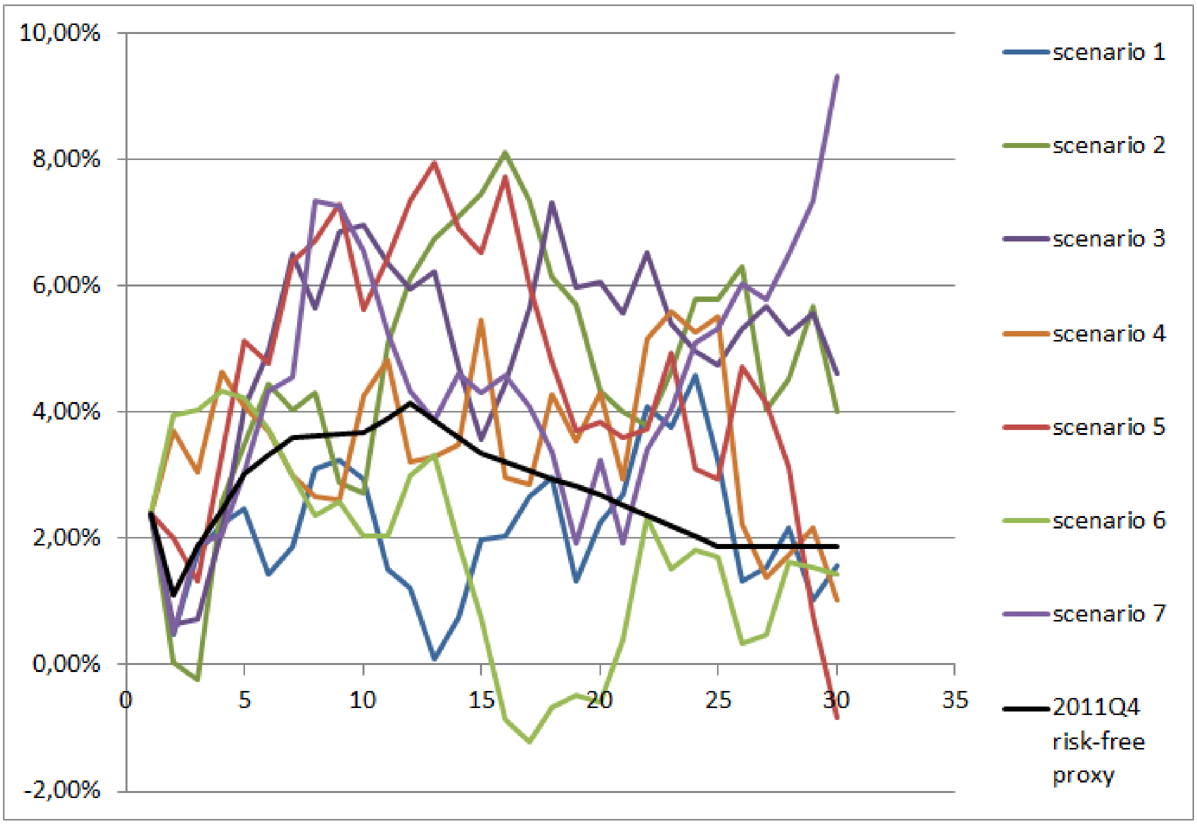

Figure 2.1: A risk-free term structure (forward one year rates) at 2011 year-end to discount life insurance liabilities, and seven risk-neutral scenarios obtained using a Hull-White model based upon this term structure. $\alpha = 5.08\%$, $\sigma = 1.21\%$

attainable contingent claim.[18] This means that if the contingent claim is replicable,[19] its price is uniquely determined by this unique replicating strategy. Not every contingent claim is attainable. A market where every contingent claim is attainable is said to be complete. If this is not the case, the market is called incomplete. Furthermore, it can be shown that an arbitrage free market is complete if and only if there exists a unique equivalent martingale. In an arbitrage-free complete market, arbitrage prices of contingent claims are their discounted expected values under the risk-neutral (unique equivalent martingale) measure $\mathbb{Q}$.

2.2 Finance theory applied to the world of insurance

The above theory has made its way to the insurance valuation world only about ten years ago. Stochastic models currently have a wide range of applications in insurance [16], most notably

[18] A contingent claim is a contract of which the payoff depends on the outcome of a stochastic variable. One of the most basic examples is a stock option, of which the payoff depends on the value of the stock, but the definition applies to a lot of financial or insurance products and portfolios.

[19] I.e. the contingent claim is attainable in the market by using other financial instruments.

in embedded value techniques, in solvency capital requirement calculations, and in financial reporting standards. The first of these two are discussed in the next two subsections, while financial reporting standards are out of the scope of this master paper.

2.2.1 Embedded Value

A natural exercise for any company would be to calculate its value. Valuation models have existed for quite some time. A basic model using the discounted values of dividends a company will pay out was developed by Gordon and published in 1959 [20]. In short, a dividend discount model uses a best-estimate of the dividends and discounts this using a return that investors expect to get from the company. This rate is thus company-specific [21]. This is a reasoning from an investor point of view, who values his share by discounting the expected future dividends.

For insurance companies, according to O'Keeffe *et al.* [22], Anderson [23] described methods to value the in-force activities[20] of an insurer also already in 1959. When a life insurer tries to calculate its own value using some kind of discounted version of its cash in- and outflows, the insurer is calculating its *embedded value*.

In calculating a Traditional Embedded Value (TEV), which was the most established way to calculate embedded value figures up until the beginning of the 2000s, one would pursue a strategy which is a lot like the dividend discount model of Gordon, but now discounting the best-estimate of the post-tax earnings, thus taking an insurer point of view, instead of dividends. Much like the dividend discount model, TEV also uses a flat *risk discount rate* to discount these cash flows. There are quite some issues with a TEV approach. First of all, the choice of the risk discount rate has always been central to the determination of the TEV. The subjectivity involved in the choice made it rather debatable [22] and non-transparent. Secondly, using this methodology, one clearly can not get a good estimate of the cost of the options inherent in the portfolio. To see this, it suffises to say that if in the best-estimate scenario, a certain very low probability, but high impact event, doesn't realise, it would be completely ignored in the valuation. Of course one can try to remedy this by making approximations, but in the light of the discussion of the previous section, it is clear that a stochastic analysis is needed to correctly value the option characteristics which are definitely present in life insurance portfolios, a trivial example being profit sharing benefits.

In 2004, the CFO Forum of European Insurers (CFO Forum)[21] issued the European Embedded

[20]The in-force activities include all policies written by the insurer at the moment of valuation. It thus excludes the value of future business, though one might argue that this is also part of the shareholders value. A discussion of the contract boundaries, which determine whether a contract should be taken into account in the valuation or not, can be found back in [24]. The discussion on the contract boundaries between the industry and EIOPA for Solvency II purposes is still ongoing, with a possible widening of the contract boundary being considered by the Draft Implementing Measures [25].

[21]The CFO Forum is a high-level discussion group formed and attended by the Chief Financial Officers of major European insurance companies.

Value (EEV) principles to create some transparancy and uniformity in the at that time quite diverging methodologies used to calculate the embedded value of life insurance companies [24]. In current embedded value reporting, most large European insurers adhere to either the EEV or the Market-Consistent Embedded Value (MCEV) Principles [26].[22] In both of these frameworks [27, 28], a stochastic valuation approach is required.

As already mentioned though, a stochastic model, be it risk-free or real-world, can only arrive at a unique value if the market is complete. It is well accepted that when valuing insurance liabilities, part of these liabilities is non-hedgeable, or in other words not attainable, which means the market is incomplete [29, 30]. This is why in the MCEV principles, the Value In Force, which is the value of the portfolio of liabilities and the assets in front of these liabilities is split into three parts:

- The stochastic value of business, which is obtained using a stochastic model, be it risk-neutral or real-world, and where the risk-free rate thus plays a crucial role;
- The Cost of Residual Non Hedgeable Risk (CRNHR), which includes the impact of the fact that the liabilites cannot be fully replicated;
- The fricitional cost of required capital, which is an extra item to take into account the taxation and investment costs on assets which are kept to constitute the required capital.

2.2.2 Solvency Requirements

Solvency II is the solvency regulation which will be applied shortly in the European Union.[23] Solvency II aims to obtain solvency of the insurer with a 99.5% probability on a yearly basis. One of the ways to obtain this, is that the available capital is sufficient to cover with a 99.5% probability shocks that might be incurred. The available capital is measured by the Net Asset Value, which is the difference between the fair value of the assets and the fair value of the liabilities. The shocks are calculated as shocks on this Net Asset Value, and these shocks determine the Solvency Capital Requirement. As already mentioned in chapter 1, the fair value approach as laid down in the Solvency II Directive more or less comes down to the fact that assets should be valued using a mark-to-market approach, while liabilities should be valued using a mark-to-model approach.[24] As the model should be market-consistent,[25] a risk-free rate is again a crucial input.

[22] The MCEV principles were published as the new standard - i.e. the successor to the EEV - in 2008 and an update was published in 2009. It was originally the goal that from end of 2011 on, reporting under MCEV would be the only accepted format for embedded value reporting. Later, the CFO Forum decided to withdraw this deadline to not impede developments in Solvency II [24].

[23] Please refer to appendix B for a brief history of solvency requirements in Europe, for the key Solvency II Directive articles related to the valuation of the liabilities, which are referred to in this section, and for an overview of the organisation of Solvency II thus far.

[24] Note that it was also mentioned in chapter 1, that for some assets, no market might be available. In such a case, a mark-to-model approach is necessary as well. This might e.g. be the case for mortgages.

[25] See Article 76 in the Solvency II Directive.

It is undisputable that there are a lot of similarities between the valuation approach required by the MCEV Principles and the approach required by the Solvency II Directive. Although we will not go into details in comparing both frameworks, note that Article 77 of the Solvency II Directive mentions that Solvency II, much like the Market-Consistent Embedded Value (MCEV) Principles, acknowledges that there needs to be an adjustment for the fact that the model can not give an exact value of the liabilities, since part of the liabilities can not be hedged. Therefore, the market is incomplete and an extra adjustment is needed. While this adjustment is termed the CRNHR in MCEV, in Solvency II it is called the risk-margin.[26]

Also in the Swiss Solvency Test, the solvency regulations to which Swiss insurers have to conform since 2006, a market-consistent approach is required [31], making a risk-free rate also for the SST an important input parameter.

2.3 Pros and cons of market-consistent value as fair value

There are clear advantages to the use of market-consistent valuation, both in solvency regulations and as an embedded value approach. First and foremost, it is a risk-sensitive approach, in contrast to the approach required by the currently applicable solvency regulation in the European Union. Because of this risk-sensitivity, it should lead to better asset-liability management, solvency management, product development and risk management in general. Furthermore, the approach is less arbitrary, has a theoretical justification and is thus harder to bypass using “smart accounting”. In embedded value calculations, the market-consistent approach introduces a uniform framework, without the need of introducing a subjective risk discount rate determination.

Calculating a market-consistent embedded value never had severe opponents, since publishing an MCEV is not required by a regulator. Therefore, critics could just disregard the “science fiction” numbers if they did not believe them. But with Solvency II coming up, science fiction will become reality. Although using a market-consistent value as a fair value for solvency purposes as a whole has been formalized in the Solvency II Directive nowadays,[27] there are and have been quite some critics of using market-consistent approach, most notably in the accounting field.[28]

In 2008, insurer AIG experienced a 11 billion USD write-down in the fair value of its holdings, leading its CEO to say that fair value accounting had unintended consequences and to demand

[26]Furthermore, the Risk Margin in Solvency II is calculated using a cost-of-capital approach, i.e. by calculating the present value of a cost-of-capital percentage times the Solvency Capital Requirement for non-hedgeable risk.

[27]Fair value and market-consistent value have practically become synonyms.

[28]The new International Financial Reporting Standard (IFRS) which is under development for insurance companies, also uses a market-consistent approach. Also in IFRS applicable to banking rather than insurance, and in the Basel II and Basel III regulations, market-consistent approaches are required. The discussions on the sense or nonsense of market-consistent techniques can however often be directly applied to Solvency II.

its suspension. Also insurer Axa stated that marking to market was a conceptual mistake, given that the assets were held against very long-term liabilities [32].

Several critics have argued that the use of market values in balance sheets is flawed on conceptual and philosophical grounds. Although most of these critics tend to have in mind the influences on banks rather than insurers, their concerns can be mostly applied to both.

2.3.1 Pro-cyclicality

One of the main issues raised by these critics is the claimed pro-cyclical consequences of fair value accounting [32, 33]. They argue that fair valuation is a utopian concept based on the efficient market hypothesis. Because there is irrationality in the market and markets are not always right, market-consistent accounting intensifies the sense of euphoria in market upswings while depressing prices in downturns. Some even go as far as to claim that fair value accounting caused or at least aggravated the financial crisis of 2008, although others oppose this claim [33, 34].

The two arguments why market-consistent valuations contribute to pro-cyclicality are the following [35]:

- When market values of assets are rising, more capital is available. This creates more opportunities to take risk, making the system more vulnerable. Using e.g. a historical cost accounting system as an alternative, creates hidden reserves for in bad times.
- When a financial institution is forced to fire sell assets at a lower value than its fundamental value,[29] this value becomes relevant to other financial institutions, which might again trigger the sale of assets.

Although there is no evidence that people would be calmer under a historical cost accounting [35], in case of fair value accounting, investors might indeed be forced to take action because of regulatory requirements, the latter being the case in Solvency II.[30] But even then it is not clear whether historical cost accounting would lead to a more healthy situation. Indeed, one of the very reasons of introducing a fair value approach is that financial institutions using a historical accounting approach might be tempted to create another problem. They might be tempted to sell assets with a market value above book value, to be able to have earnings and create a nice outer image, while keeping low market value assets in the portfolio and thus their value in the books at a higher than market value. This way no hidden reserve is constituted, leaving

[29] It is unclear what exactly is the fundamental value. But it is imaginable that when an asset, particularly when it is not very liquid, has to be sold quickly, one might be paid a lower price then when the asset does not necessarily need to be sold quickly.

[30] On top of this, if in stressed markets the financial situation of the insurer is weaker, this is directly visible in its solvency position, because of the risk-sensitivity of the market-consistent approach. Because of the visibility of this weaker position, surrender rates might increase, forcing insurers to sell illiquid assets at a lower price then the value it has to the insurer in case it is held to maturity [36].

outsiders with only intransparant numbers leading to even more questions. Using the same type of reasoning, Laux and Lenz [35] acknowledge the potential irrationality and the pro-cyclical behaviour, but they conclude that the historical cost alternative is definitely not better.[31]

Nevertheless, potential pro-cyclical issues need to be carefully addressed. The aim of Solvency II is first en foremost to bring a better protection to policyholders. A regulation which might cause or aggravate insolvency is definitely not the objective. Because historical cost accounting is not a good alternative, De Grauwe, although being one of the critics mentioned above [33], advocated for a market-consistent approach, but using measures to introduce inertia in market prices. A similar reasoning is found back with Laux and Lenz who advocate the inclusion of counter-cyclical capital requirement measures.

2.3.2 Illiquid market

Another issue which is of importance for insurance, is that finance theory, as introduced in section 2.1, assumes deep, liquid and complete markets, which is definitely not the case for insurance liabilities. This makes the applicability to insurance liabilities rather controversial [37]. To my opinion however, if one acknowledges the incompleteness of the market and incorporates adjustments such as a risk margin, then it is not necessary for insurance liabilities to be deep and liquid in the sense that they can be traded easily *between insurers* to determine a market-consistent price. However, the illiquidity of the liabilities due to the fact that insurance contracts are not liquidly tradable between policyholders does pose some methodological problems. This problem is further dealth with in chapter 5.

Moreover, a similar but distinct issue raised is that fair value accounting assumes liquid markets [32, 33], where liquidity tends to dry up in times of financial stress [38]. This means that a market-consistent approach in times of crisis is inherently unapplicable.

In spite of these issues, nowadays few critics go as far as to say that finance theory is not to be applied to an insurance context whatsoever. As mentioned, the Solvency II Directive already mentions the requirement of a market-consistent approach. There is not so much criticism on a market-consistent approach as a whole, but rather concerns how to apply the finance theory to an insurance context, while remaining market-consistent, taking into account possible illiquidity characteristics. As mentioned, we will not discuss this issue in this chapter, but will postpone it to chapter 5.

[31] In case of a bank, De Grauwe possibly rightly argues that illiquidity issues might be aggravated pro-cyclically in case of market-consistent valuation [32]. This master paper however is written with an insurance mind set and an insurer mostly needs to worry about its solvency, not about its liquidity position.

2.4 Conclusion

Risk-free rates are an important input parameter in pricing models which are used extensively in finance. These models, which in the past have been mostly used to price contingent claims, have currently made their way to the valuation of insurance liabilities. A market-consistent value of the company is calculated by taking the market value of the assets and substracting the modelled market-consistent value of the liabilities, thus obtaining the Net Asset Value. This approach has been used in embedded value reporting for quite some time now and as it has made its way into Solvency II, the new European solvency regulation, the choice of the risk-free rate might have a huge impact on the capital insurers are expected to hold.

Furthermore, the choice of the risk-free rate in an insurance context is particularly important because of the often long-term nature of the liability portfolio, which introduces an even stronger dependency on the discount rate used.

Finally, although using a market-consistent approach introduces a risk-sensitive measurement approach, it might introduce pro-cyclicality. To counteract this unintended side effect, it might be necessary to introduce counter-cyclical measures in Solvency II, but it has to be kept in mind that the current Solvency II Directive explicitely requires a market-consistent approach.

Chapter 3

The quest for a risk-free rate

Chapter 2 explained why the choice of risk-free rates is so important in the current life insurance environment. This chapter discusses different risk-free rate candidates, and searches the best risk-free rate for life insurance liability valuation. In section 3.1, a theoretical and widely accepted decomposition of market rates is presented. One of these components is the risk-free rate, which leaves the question which rates should be used as risk-free rates in life insurance valuation. This question is explored in section 3.2. In section 3.3, it is concluded that swap rates are currently probably the best candidates, for multiple reasons. However, the amount of credit spread in swap rates might be a lot higher than currently accounted for in embedded value and Solvency II calculations.

3.1 What is a real rate composed of?

The risk-free rate mentioned in most introductory finance models is said to be a risk-free bank account. Of course it makes no sense as an individual to go to the bank office around the corner, and ask what the return on their bank account is, and then to use this as risk-free rate. First of all, this rate is not risk-free, since the bank can default, and secondly, the bank will probably take a margin, so the rate will probably be lower than the risk-free rate a typical investor can get on the market.

A better idea would be to use a rate which is openly published and thus available to anyone. One then has to pay attention to the fact that the risk-free rate required in a valuation model will have a term structure, so the risk-free rate will probably be different for different maturities. It is important to clearly understand the differences between the way rates for different maturities are quoted. Distinction has to be made between zero-coupon rates, forward rates, and par yields. If the reader is not familiar with one of these terms, we suggest to consult appendix A.

Theoretically, a rate, say an investment return on a bond, can be divided into the following components:

- A risk-free return,
- a credit spread,
- a liquidity premium, and
- a remainder.

The credit spread is related to the default risk, i.e. an investor requires a higher return if there is a risk that the counterparty might default. A liquidity premium is a compensation for an investor investing in an illiquid asset, since should the investor ever be forced to sell the asset quickly, there is a risk that the investor will receive a lower price than the expected value of the run off of the asset. The remainder contains a number of different components that are difficult to quantify and disentangle [39]. Some argue that the remainder contains an (irrational) non-fundamental credit spread, which is the difference between a credit spread corresponding to the probability of default and the total credit spread.[1]

It is important to note that the split made above is mostly a theoretical construction, in the sense that the market will only show us the total rate, and assumptions have to be made to split up the rate in different components. One might for example argue that holding an illiquid asset together with an associated Credit Default Swap (CDS) takes away the credit risk component, leaving the liquidity premium and the remainder [39]. This approach entails a number of issues, most notably the issue that in a CDS, there is also a credit risk, which is perhaps even correlated to the default of the bond issuer.

3.2 Finding a risk-free rate for insurance liability valuation

When one is looking for a risk-free rate, the number one criterion that is taken into account is the absence of default risk, since finding assets with a low illiquidity risk is mostly not that much of a problem, while finding a default-risk-free asset is a little harder.[2]

Most general articles discussing the issue of which risk-free rates to use, tend to look for risk-free rates for maturities up to one, maybe five years. When we look at the liabilities of a typical life insurance portfolio, it is clear that risk-free rates are needed up to thirty or fifty years.

[1] There is indeed quite some empirical evidence that default rates have historically been below the credit spreads [40, 41], a problem termed the *credit spread puzzle*. However, this is only a problem if one does not make a distinction between real-world probabilities and risk-neutral probabilities. If all investors were risk-neutral, indeed default rates would be equal to credit spreads. But investors are typically rather risk-averse, thus demanding an additional risk premium on top of the default probability.

[2] Note that we do not consider the absence of inflation risk as a criterion, since the risk-free rate needed for pricing using no-arbitrage arguments does not need to be free of inflation risk. We also don't consider reinvestment risk, as we suppose that the risk-free candidate rates discussed below are rates that can effectively be earned in the market. We thus assume that the forward rates implied by todays term structure can be locked in today. This assumption, which is inherent in most pricing models, eliminates reinvestment risk.

3.2.1 Government bond rates

In the current economic environment, it might be hard to imagine that there is truly an asset which has no default risk. Nevertheless, one might argue that e.g. US Treasury bills, bonds and notes are free of default risk, since the US government, at least in theory, could always pay its debts, simply by printing money [42, 43]. Note that this reasoning does not hold for European government bonds. It does however hold for Swiss bonds.

However, even for US Treasury rates, there are multiple reasons why they are not a very good choice as a risk-free rate, as they tend to be lower than other rates that have very low credit risk for a number of different reasons [41]. Treasuries receive e.g. a favourable tax treatment in the US, and the amount of capital a US bank is required to hold is much lower when it is invested in Treasuries than for similar other investments, which increases demand.

For European government bonds, not only is the above money printing possibility theoretically absent, also in practice most European government rates have shown considerable spreads over the last year. Government bond rates are thus not much of a candidate for use as a risk-free discount rate in the context of Solvency II or Embedded Value reporting. One could think about opting for a selection of highly rated (say AAA) European government bonds, but this might not be a long-term solution, as it is not unimaginable that at some point in the future, no government bonds will still be highly rated. Furthermore, these highly rated bonds might have similar issues as the US Treasury just mentioned.

Although nowadays it is rather clear that using European government rates is not a good idea, until the beginning of 2008, many Belgian insurers were using the Belgian OLO[3] curve to discount liability cash-flows in many regulatory applications [44]. They did so for historical reasons or to ease communication with regulatory authorities, which had been using OLO rates for other topics, such as the maximum technical rate for certain insurance products (Article 24 in the Royal Decree concerning life insurance[4]). At that time, there was not much of a debate on the topic, since government curves were not too far from other risk-free rate candidates, such as the swap curve. This is illustrated in figure 3.1, which shows the evolution of the 10 year maturity government bond rate over the last decade and compares it with swap rates.

Regulators have been going back and forth between choosing the government yield curve or the swap yield curve (see further) as risk-free rates. Within Solvency II, according to Sender [45], *before* QIS4,[5] which was held in December 2007, CEIOPS[6] still recommended using government

[3]OLO stands for Obligation Linaire/Lineaire Obligatie, and is a common name used for Belgian Government Bonds.

[4]Royal Decree of November 14, 2003 on life insurance activities.

[5]QIS4 was the fourth Quantitative Impact Study which was organized within the framework of Solvency II, see appendix B.

[6]CEIOPS was the Committee of European Insurance and Occupational Pensions Supervisors and is now replaced by EIOPA, the European Insurance and Occupational Pensions Authority.

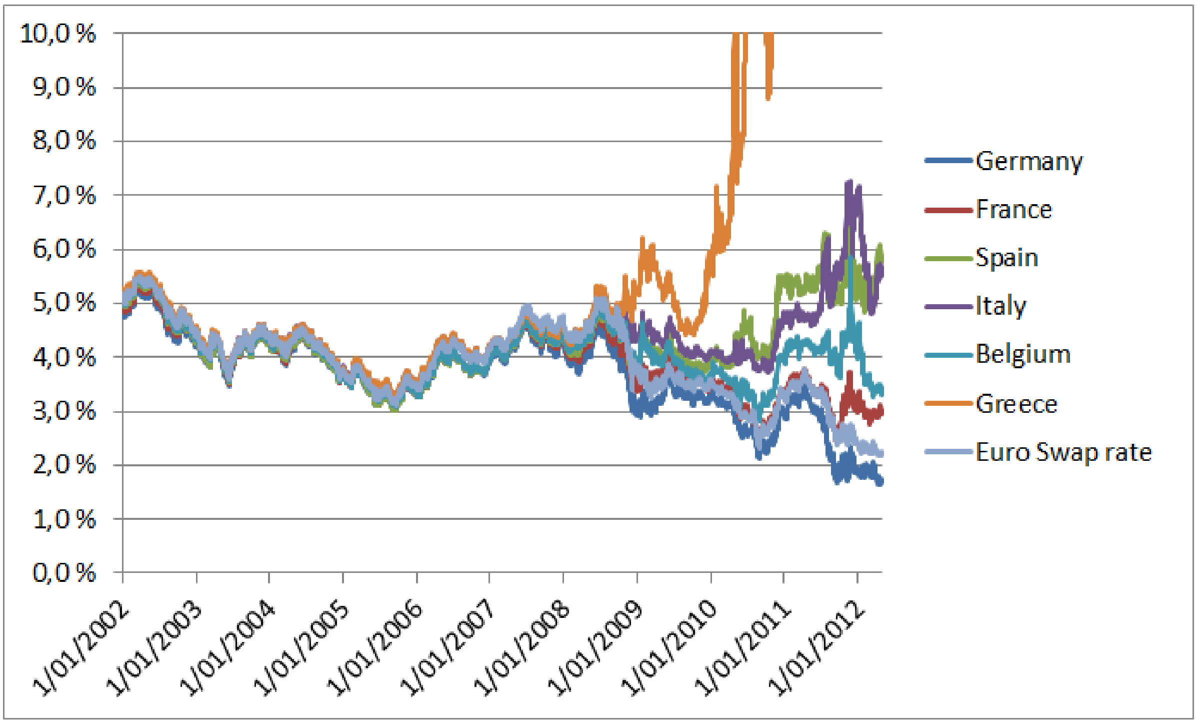

Figure 3.1: Evolution of 10 year maturity government bond rates compared to 10 year swap rate. Source: Bloomberg.

yields. But even then, looking at the QIS4 technical specifications [46], CEIOPS specified:

> For QIS4 purposes, the prescribed risk-free interest rate term structure for cash-flows denominated in Euro is the Euro area yield curve published by the European Central Bank (ECB).

The yield curve was based on AAA-rated European government bonds.

The QIS5 Technical Specifications stopped prescribing government bond rates as risk-free rates, see section 3.2.3.

Furthermore, it is remarkable that in the Swiss Solvency Test, the Swiss government bond rate is currently still prescribed as the risk-free rate [36].

Nevertheless, in a European context as we currently know it, it should still be clear that using government rates is not an option. Alternatives need to be sought.

3.2.2 Interbank Rates

Hull [6] argues that most financial institutions have traditionally used LIBOR as risk-free rates. LIBOR is the London Interbank Offered Rate. It is the interest rate that a selection of banks in London charge to each other when one lends to another. On a European level, one could thus think about using Euribor[7] as a risk-free rate. This is a logical choice in the sense that it is these financial institutions' short-term opportunity cost of capital.

There are some problems with LIBOR and Euribor though. First of all, they are not default free. Especially since 2007, banks became very reluctant to lend each other during the crisis and both rates roared in 2008, which can clearly be seen on 3.2. An even bigger problem in an insurance context is that LIBOR and Euribor are only available for maturities up to 1 year, which makes them unsuitable for use in a life insurance context.

A final, less often mentioned, issue with both LIBOR and Euribor is that they are the rate at which banks *tell* the British Bankers' Association (for LIBOR) or Reuters (for Euribor) they feel they should pay to borrow. It is not based on real interbank rates, which potentially leads to manipulation issues [47]. One could think of using EONIA[8] rates [48], which are determined using real overnight realised transactions. The problem is then of course that, EONIA rates are only overnight, i.e. one day, rates and are not available for any term, which makes them even more unsuitable than Euribor for use in a life insurance liability valuation model.

3.2.3 Swap rates

Market participants lately tend to regard swap rates as better proxies for risk-free rates [41, 6]. An interest rate swap is a contractual agreement between two parties to make periodic payments to one another based on a fixed rate and a floating rate. The fixed rate is then the *swap rate* and the floating rate is usually an interbank rate, such as LIBOR [49].[9] For an Overnight Indexed Swap (OIS), in which the geometric average of overnight rates is used as the floating rate, mostly rates set by central banks are used. These rates are often targeted rates to influence monetary policy [6]. As already noted, the CDS market provides a way of estimating the benchmark risk-free rate used by participants in credit market. Using this type of analysis Hull *et al.* estimated already in 2004 that the benchmark risk-free rate being used by the short-term USD bond market is the USD swap rate less only 10 basis points [50], based on a data sample, consisting of 964 short-maturity[10] USD bonds issued by 31 reference entities from 1998 to 2002.

In the following sections, we shall discuss the particularities of swap rates, while focusing on the aspects that make them good risk-free rate candidates. Firstly, section 3.2.3.1 explains why swap rates typically contain a relatively low credit spread. Secondly, section 3.2.3.2 argues

[7] Euribor is the Euro Interbank Offered Rate. It is the European equivalent of the LIBOR, which means that it is the interest rates that a selection of European banks charge each other when one lends to another. As can be see in figure 3.2, Euribor and LIBOR (euro) rates have always been very close to each other.

[8] Euro OverNight Indexed Average.

[9] In fact, to be able to effectively *earn* the swap rate which is quoted in the market, one needs to enter into sequential loans, which will be at the floating rate, and then swap the interest payments for fixed payments with the swap contract. Therefore, swaps cannot constitute a replicating portfolio by themselves. A floating/short rate should be added to the market to be able to earn the swap rates [44].

[10] 5 year maturity.

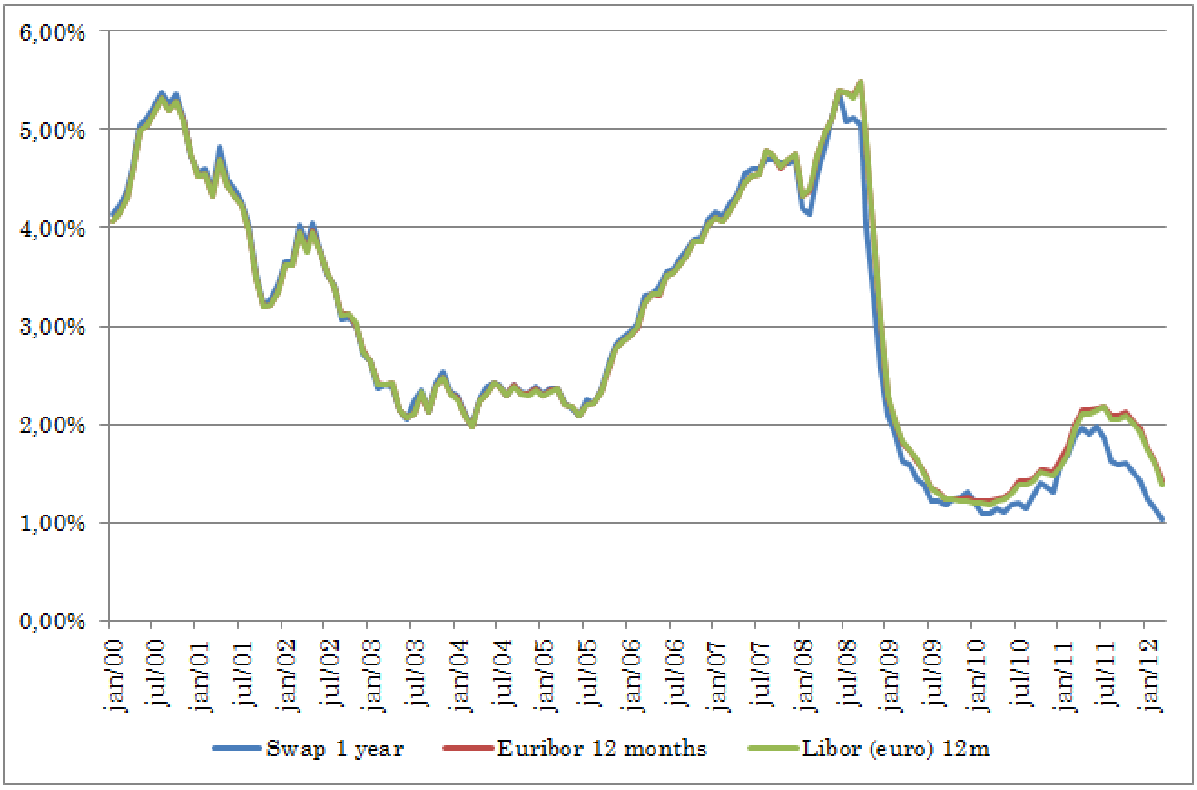

Figure 3.2: Comparison of euro swap rates, Euribor and LIBOR (euro) over the past decade. Source: Bloomberg.

that credit risk is by far the largest contributer to the spread between a swap rate and a truly risk-free rate, because of high liquidity. This holds even though some researchers argue that swap rates do contain a liquidity premium, but this is based on a misunderstanding, as we shall argue in the next section. We do note that there might be illiquidity issues for swap rate maturities higher than 20 or 30 years however. Finally, to conclude the section on swap rates, section 3.2.3.3 discusses the current use of swaps in an insurance valuation context, highlighting a probably far too low estimation of the credit spread which is currently part of the swap rate.

3.2.3.1 Credit risk

It is often argued that the credit risk on a swap *contract* is lower than on an interbank loan [51, 52]. This is true, because of the fact that in a swap agreement, there is no exchange of nominal amounts. Furthermore, although swap contracts are mostly OTC contracts [53], and thus without guarantee of a clearing house, swaps are rarely entered into by two regular companies [6]. Instead, a company enters into a swap with a financial intermediary. The latter can, by entering into other swaps with other counterparties, construct a portfolio on which he can gain a credit spread, which is charged because the companies with whom he enters into the swaps can default. The financial intermediary can use a system of margins or collateral to protect himself against the counterparty default risk. This in turn reduces the credit risk on a swap transaction.

It is however wrong to *therefore* conclude that swap *rates* contain a smaller credit spread than interbank rates.[11] As already mentioned, to be able to earn the swap rate, a company needs to enter into a sequence of loans at the floating rate, swapping the floating rate payments he earns for the fixed swap rate. Because of this, any financial institution who wants to earn the swap rate, needs to engage in interbank loans, at a rate which contains credit risk. Therefore, the swap rate inherently also contains a credit spread. This can be shown by the following example.

Suppose the Fantasy Interbank Offered Rate (FIBOR) at 6 months, is at 5%, of which 4% is the risk-free rate, and 1% is due to credit risk. The swap contract which swaps two times the floating FIBOR against two fixed rate payments, one in six months from now and one in a year, specifies fixed rate payments of 6%. There is no default risk in the swap contract whatsoever. Let us further assume for simplicity reasons that the credit spread in the FIBOR remains unchanged at 25% of the risk-free rate. To earn the swap rate, a financial institution has to enter into two sequential six months loans at the FIBOR, for which he will receive 5% in six months from now and an unkown payment $x\%$ in twelve months from now. Suppose that as an alternative, the financial institution has the possibility to enter into two sequential six months loans at the risk-free rate, for which he will receive 4% in six months from now and an unknown payment $0.8x$ in twelve months from now. Because of the proportionality, this contract can be swapped using the same swap contract, the only difference being that the contract is only entered into for 0.80 of the original notional amount. Therefore, a swap contract using the risk-free rate as the floating rate, will pay $0.8 \times 6\% = 4.8\%$ as the swap rate. It is clear that the credit spread in the swap agreement using FIBOR thus contains a spread which is due to the credit risk in a FIBOR loan.[12]

Nevertheless, in practice e.g. a 1 year interbank rate *will* be lower than the 1 year swap rate. In fact, the annual swap rate one usually uses for euro in a Solvency II context,[13] which is the one depicted in figure 3.2, is based upon swapping floating 6 month Euribor payments for fixed yearly payments. Looking at figure 3.2, there is indeed a difference between a 12 month Euribor and the 1 year swap rate. This difference can be explained [50] by the fact that there is less credit risk in lending two consecutive times for six months to a highly rated financial institution,[14] than lending one time for twelve months to a highly rated financial institution, since after six months, the health of the counterparty can be reassessed and if necessary, the

[11] Although they, but do for another reason, see further.

[12] This example also demonstrates that if Euribor is misquoted because of the fact that it is based on a survey rather than on real transactions, this directly influences the swap contracts which use Euribor as the floating rate.

[13] Bloomberg ticker EUSAX, where X is the maturity.

[14] Only the highest rated financial institutions pay Euribor or LIBOR without a credit risk premium.

money can be leant to another, higher rated counterparty for the remaining six months.

One can take this reasoning several steps further, and split up the term in a sequence of one day loans, which is what is done in an OIS. The total credit risk is then the sequence of one day credit risks, which is extremely low. The spread between the LIBOR and the OIS,[15] is therefore often used as an indicator of the health of the banking system in the sense that it reflects how likely banks will default [52]. As one can see in figure D.2 in appendix D, the spread here is much larger than the one illustrated in figure 3.2, because of the extremely low credit risk in the OIS.

Resuming, interbank rate based swap rates tend to have lower credit risk than the interbank rate itself, and overnight indexed swaps tend to have even lower credit risk.

One could therefore think about using swap rates based on EONIA rates as the floating rates, i.e. an EONIA based OIS. The problem with EONIA swap quotes is that although EONIA swap contracts are very popular for short term swaps,[16] Euribor remains the key reference rate for longer term swaps [54]. This means that for insurance application, swaps based on Euribor are currently the only option. An additional disadvantage with EONIA swap rate quotes is that they are again based on a survey, rather than on real EONIA swap contracts [55].

3.2.3.2 Other risk premiums

According to Chung and Chan [49], *swap spreads* are jointly determined by a liquidity premium and a default risk premium.[17] Grinblatt proposed already back in 2001 that swap spreads are compensation for the higher liquidity associated with holding government securities as compared to entering into a swap contract to receive fixed payments [56], hence the term liquidity premium (see also next chapter). Using different proxies for the liquidity premium, a variety of researchers[18] find that *variations* in swap spread are mainly caused by changes in the liquidity premium.

In my opinion, the liquidity premium discussed here is a little misspecified. The swap spread is defined [49, 57] as the difference between the (USD) swap rate and the Treasury rate. As already mentioned above, Treasuries are not a good candidate for a risk-free rate. This means that the swap spread is not the spread on top of the risk-free rate. The interest rate swap market is the largest over-the-counter (OTC) interest rate derivatives market, in general, and the euro interest rate swap market is one of the most liquid financial markets in the world [54]. A liquidity premium on top of the risk-free rate might thus be hard to justify. The same idea is

[15] Both USD.

[16] And since 2000 it even supersedes Euribor swaps at the short end of the swap curve.

[17] They also mention the interest rate level and the business cycle, but then go on to argue that swap spread is a form of credit spread, and that credit spreads are in general influenced by interest rate levels and the business cycle.

[18] For a list, see Chung and Chan [49].

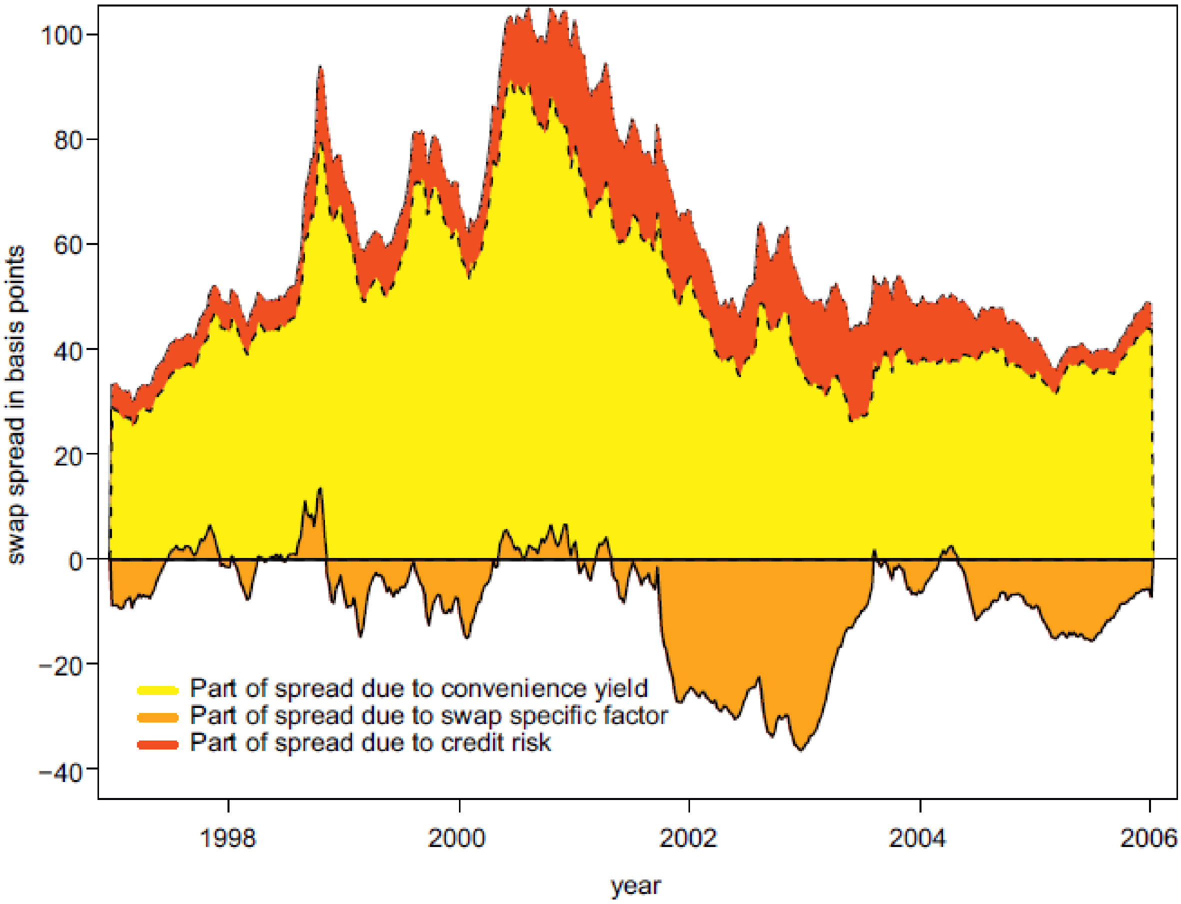

Figure 3.3: Decomposition of the ten-year spread between Treasuries and swap rates. Figure taken from Feldhütter and Lando, for details on determination of breakdown, see [59]

partially put forward by Liu *et al.* [58], who admit that because of the extreme liquidity of Treasury bills, Treasury yields tend to underestimate the effective riskless rate. An even more specific treatment of this issue is performed by Feldhütter and Lando [59], who argue that the size of the *convenience yield* instead of a liquidity premium is what separates the Treasury yield from the risk-free rate and it is by far the largest contributing factor to the swap spread.[19] The USD swap rate is separated from the risk-free rate by a relatively stable (LIBOR) credit risk component, and by a swap specific factor. They go on to separate the three components, and their results are illustrated in Figure 3.3.

We could thus conclude that one of the most important advantages of using swap rates as a risk-free rate proxy, given that a correction for the credit spread is made, is that swaps[20] are very liquid [60]. This avoids the need to cancel out liquidity premia, which are inherently

[19]The swap spread is again defined as the difference between Treasury yield and the USD swap rate.
[20]As well as the interbank loans needed to earn the swap rate.

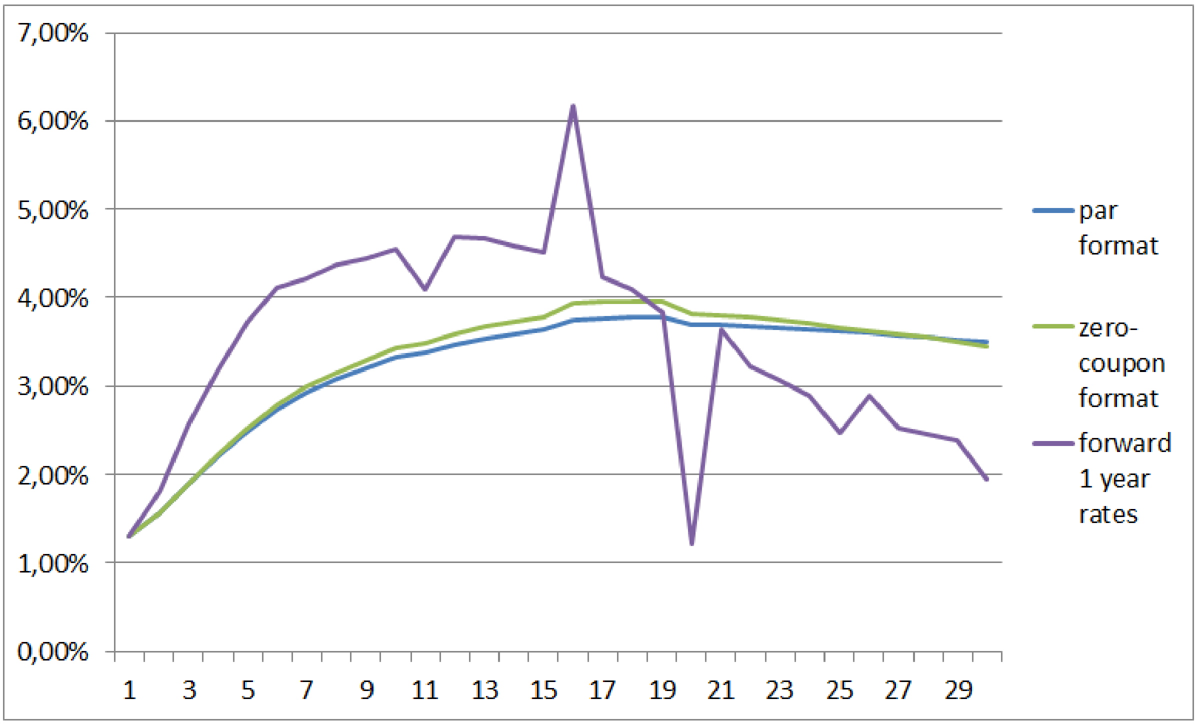

Figure 3.4: The euro swap rate term structure at 30 December 2010, quoted in different formats. Source of rates in par format: Bloomberg.

unreliable, because of the price uncertainty that goes along with illiquidity, see also chapter 5.

However, as mentioned, an insurance portfolio might contain liabilities with maturities up to 50 years and higher. Therefore, risk-free rates are needed for very high maturities. It is often argued that swap rates are not liquid after a certain point in time, which is estimated between 20 and 30 years for euro swap rates.[21] We will not deal with the issue of what needs to be done after the last liquid point in the scope of this master paper, although the issue is interesting, challenging and has a huge impact on long-term insurance business.

Apart from the possible illiquidity issues after a certain term, a second, less important issue is that swap rates are possibly less liquid for some maturities, leading to a strange behaviour of the forward rates derived directly from the quoted swap rate, as illustrated in figure 3.4, which shows the forward 1 year rates derived from the euro swap rate as quoted on Bloomberg end of December 2010, using the formulae mentioned in appendix A.

To the extent that this is really considered an issue,[22] it can be solved by a smoothing

[21] For QIS5 purposes, where swap rates adjusted for credit risk were used as the basic risk-free term structure, see further, the rates were extrapolated instead of based on market data from 30 years on for euro [60]. Discussions are still ongoing on what extrapolation method to apply, what the Ultimate Forward Rate (UFR) should be, and how fast convergence should take place.

[22] Extreme forward rates might render a model less stable, and there is generally no economical justification

interpolation procedure on the forward rates, which mostly has a negligible impact on the rates expressed as par yields.

3.2.3.3 Use in an insurance valuation context

CEIOPS states [61] that the non-extrapolated part of the risk-free interest rate curve for QIS5 purposes was delivered by the industry. In practice, this meant that the method used to derive the basic risk-free rate was the one suggested by a publication of the CFO Forum and the CRO Forum [60], which is also available on the European Commission website. The basis risk-free rate used for euro was the euro swap rate with a small adjustment for credit spread. They came to the same conclusion as Hull *et al.* in 2004 for USD swap rates, namely that the credit spread adjustment on euro swap rates should be 10bp, but using another type of analysis.[23]

They used the difference between short-term (3 to 6 months) unsecured inter-bank lending (Euribor-like) and short-term (3 to 6 months) secured repurchase agreement (repo) rates as a measure for the credit risk in swap rates, assuming the latter are risk free and assuming the credit risk in the former is a good estimator of the credit risk in swaps.[24] No details on the time window are given, but they do make a split between a pre-crisis figure and a in-crisis figure.[25] The difference for euro swap rates between the pre- and in-crisis figure was estimated to be only around 1 to 14bp. It is important to note that the method used by the CFO Forum and the CRO Forum relies on the observation that the orange part in Figure 3.3 is small. Although this might be defended when looking at Figure 3.3, it would be interesting to see the impact of the financial crisis on the breakdown of the swap spread.

To make a quick and dirty estimation of the recent evolution of the credit spread in Euribor based swaps, one could compare the 6 month Euribor with the 6 month EONIA swap. The reasoning is that the credit risk on long-term swaps based on 6 month Euribor is approximately equal to the credit risk on 6 month Euribor,[26] while the 6 month EONIA swap has very low credit risk. A comparison is shown in figure 3.5. The spread is of the same shape as the (USD) LIBOR OIS spread. The spread up until the end of July 2007 is very constant and around 10bp.[27] Since then the spread stayed above 50bp most of the time, with peaks of 100 to 125bp.

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for the erratic behaviour.

[23] The analysis is based upon the one made by Li [57].

[24] As a justification, they mention "Features such as collateralisation arrangements and the fact that the notional amount is never at risk means the credit risk in a swap contract is negligible. However, for swap contracts based on 3 or 6 month inter-bank rates there can be credit risk associated with earning the reference floating rate, as there is some risk associated with depositing the notional amount with an institution for the 3 to 6 month period. [...] at this time the inter-bank swap curve represents the most liquid and therefore reliable source of data. We therefore focus on the deposit risk in inter-bank swaps [60]."

[25] The publication was published in April 2010.

[26] Assuming the credit risk on 6 month Euribor will remain stable and assuming no credit risk in the swap contract.

[27] In fact, the average starting from the introduction of the EONIA swap quote in 2005 until the end of July 2007 is only 7bp. This figure is consistent with the pre-crisis number estimated by the CFO Forum and the CRO

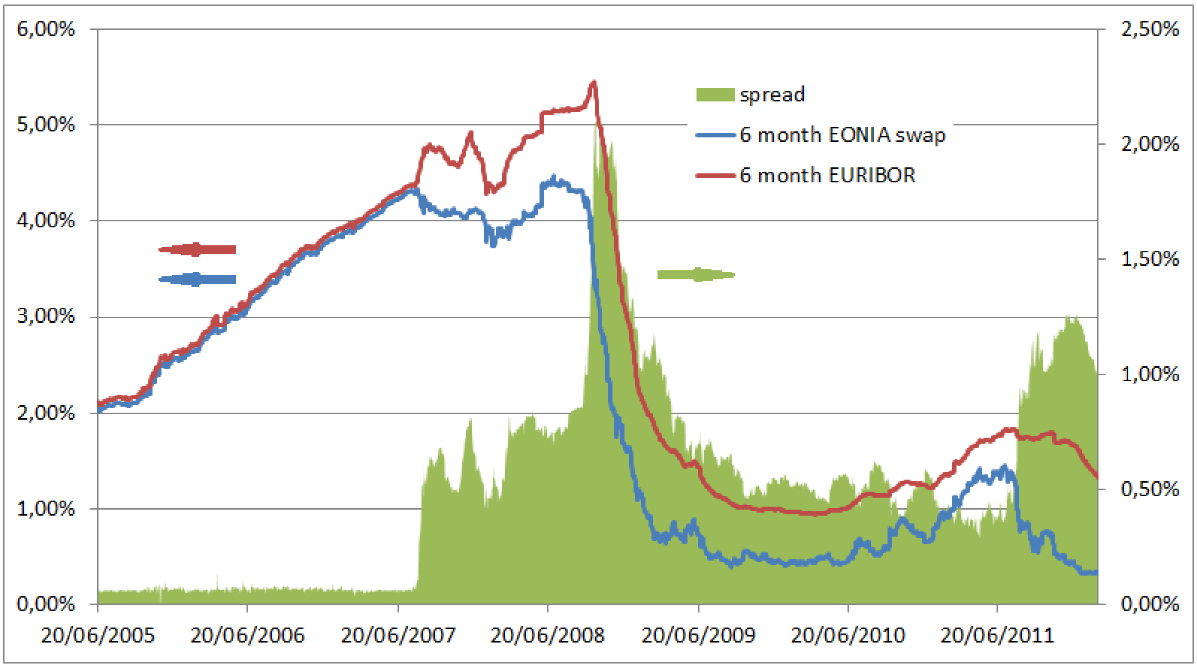

Figure 3.5: Comparison of 6 month Euribor and 6 month EONIA swap. Spread is plotted w.r.t. secondary (right) axis. Source: `www.euribor-ebf.eu`.

Although figure 3.5 does not pretend to provide a fully accurate estimate of the credit spread in Euribor based swap rates, it should definitely trigger a reassesment of the estimation of the credit spread of only 10bp used in the industry and indirectly prescribed by EIOPA through its support of the CRO and CFO Forum. It is astonishing that this estimate has not been questioned before.

3.3 Conclusion

In the quest for a risk-free rate to use for life insurance liability valuation, one needs to look for a rate with the following characteristics:

- low credit risk;
- based on liquid instruments;
- available (liquid and reliable) up to long maturities;
- external, i.e. out-of-control of the insurer.

On top of this, the rate should ideally continue to have these characteristics in perpetuum.

In considering risk-free candidates, we first discussed the possible use of government bond rates. However, because of the current European credit crisis, it is clear that most government rates in

Forum in their QIS5 Technical Specification Risk-free interest rates [60].

Europe contain a considerable credit spread. It is interesting to note that in the Swiss Solvency Test however, Swiss government bond rates are still used today as the risk free rates. Because the credit spread in todays Swiss government bond rates is rather low, this has not yet posed significant problems.

Next, we discussed the use of interbank offered rates, such as LIBOR and Euribor. The main problem with these rates is the rather high credit risk and, even more important for insurance, the fact that these rates are unavailable for long maturities. A third issue is that these rates are based on a survey held amongst banks, instead of being based on real loans. An alternative, such as EONIA, which is an overnight interbank rate as well and which is based on real transactions, and which has the advantage of lower credit risk on top of this, has the disadvantage that it is not available for any term, since it is only an overnight rate. Because of this, EONIA rates are no candidate for use in an insurance context.

Finally, the use of swap rates was put forward as the best risk-free candidate currently available for risk-free discounting of insurance liabilities. The credit risk is lower than for interbank rates, higher maturity rates are available, and the credit risk appears to have been rather stable over time. On top of this, there is evidence that the market uses swap rates adjusted for credit spread as its risk-free pricing rate.

As swaps are inherently influenced by the floating short rate on which they are based, it would seem reasonable to consider EONIA-based swaps, because of the mentioned low credit risk in overnight rates, but EONIA-swaps are only available up to 24 months, and rates with longer maturities are definitely needed. For longer maturities, Euribor is still the most commonly used floating rate for longer term euro swaps. Therefore, Euribor-swap rates are currently the best candidate risk-free rate.

Currently, the industry, academics and the regulators seem to have agreed upon the use of Euribor-based swap rates for Solvency II and MCEV purposes. Although this convergence is a good thing, and although a stable methodology is preferable, the use of swap rates in general should be questioned at least periodically, including questioning the short rates on which they are based. The methodology to determine Euribor, which is currently based on a survey rather than on real rates, creates a danger of misspecification for Euribor-based swap rates as well. One should consider a possible review of the Euribor determination methodology. Even more importantly, the credit spread should be monitored and most urgently, the impact of the most recent economic events on the credit spread should be estimated. A simple analysis made shows that this impact might be enormous, with estimations of credit spreads which are currently at about 100bp. Although this credit spread might seem so high that the use of Euribor-based swap rates should be questioned altogether, one needs to hold in mind that a better alternative is likely unavailable.

Chapter 4

Adapting the risk-free rate to insurers' needs

Life insurers hold assets to hedge the financial risk of their liabilities. To a large extent, at least for more or less classic life insurance products,[1] insurers invest in bonds to do so. Classically, an insurer was able to give an interest rate guarantee which was a little below the interest rate he could get from investing in low-risk assets, such as government bonds or highly rated corporate bonds. The margin between the interest rate on the market and the guaranteed interest rate on the insurance contract was then used to cover expenses made and also as a return (incl. compensation for risk bearing) for the shareholder.

During the credit crunch in 2008, insurers holding corporate bonds experienced a material drop in asset values due to widening corporate bond spreads [62]. Since government bonds at that time displayed still rather low spreads, investors were fleeing corporate bonds and were mainly looking to invest in government bonds. This lead to a drop in the risk-free (government) interest rate and to a large rise in corporate spreads.

Of course the impact on the value of insurance companies was twofold, as the value of their assets decreased because of widening spreads, while the value of their liabilities increased, because of the drop in the risk-free rate [39]. European insurers valuing their liability portfolios in a market-consistent manner, were therefore experiencing material drops in their Market-Consistent Embedded Values (MCEV) and hence probably in their Solvency II Net Asset Value (NAV) as well, because in general, the use of a lower risk-free rate leads, ceteris paribus, to a higher value of liabilities.[2]

These issues have recently only been aggravated by the European government credit crisis,

[1] Thus excluding e.g. unit linked policies.

[2] Although in most cases it is true that a lower discount rate means lower value, this is not always the case, most notably not for future business. See also appendix C.

since again the spreads (now on government rates) have increased, while the risk-free rates have not increased since 2008 or 2009, see figure 3.2 and the combination[3] of figures 3.1 and 3.5 in the previous chapter.

As mentioned, spreads are currently high. These spreads consist of credit risk, liquidity premia, and a remainder, see section 3.1. Since 2008, insurers have been arguing to include part of the spread on top of the risk-free rate to then use this as a tailored risk-free rate,[4] for a variety of reasons. We shall only briefly touch upon these reasons in this chapter, postponing the discussion on the theoretical justification of all of these premia to chapter 5.

Section 4.1 mentions the quick rise and acceptation of the use of a liquidity premium as required in the Solvency II QIS5. The more recent ideas, which are mentioned in the Draft Implementing Measures Solvency II, and which will likely be replacing the liquidity premium, are the inclusion of a matching premium and a counter-cyclical premium, which are discussed in section 4.2.

4.1 The liquidity premium

The liquidity premium[5] (LP) is a premium on top of the risk free rate that an investor would demand in return for the fact that an asset he is buying is illiquid [64],[6] since the investor faces an illiquidity risk, i.e. the risk that when the investor would have to resell the asset, he would have to accept a too low price because of the illiquidity of the asset.

Research by the insurance industry showed that by the end of 2008 bond spreads exceeded by far the cost of credit risk mitigation and included a new component which was much less visible in the years before: the liquidity premium [62]. The existence of a liquidity premium in current spreads is to our knowledge not contested by anyone.[7] Around the same period,[8]

[3]The former shows the decrease in swap rates, while on top of this, the latter is an indication that the credit spread in these swap rates has probably increased.

[4]An attentive reader might make the following remark: "Why are these adjustments that important? As mentioned in chapter 2, the risk-free rate is mostly used as an input to a no-arbitrage interest rate model. Since the parameters of this model are then calibrated on observed market prices, the impact of the adjustments mentioned should be greatly diminished, since with or without the adjustments, the prices should be replicated. E.g. in a Hull-White model, when a LP is added to the risk-free adjustment of the swap rate, the mean-reversion parameter and the short-rate volatility would probably be respectively lower and higher then when using a risk-free rate without LP to fit the same market data." However, "to ensure market consistency", the CRO Forum advises (for Solvency II purposes) to calibrate the models to observable market prices *before* allowance for liability illiquidity [63]. Also in the MCEV principle, Guideline 15.2 states that "The calibration [. . .] should be based on [. . .] the initial swap rate curve [. . .]".

[5]See also chapter 3.

[6]When refering to the premium in this way, it makes more sense to talk about an illiquidity premium. A liquidity premium is in fact the extra value that the market attaches to a liquid asset when compared to an illiquid asset. Nevertheless, we shall use the term liquidity premium in both cases in this master paper.

[7]I.e. its existence is agreed upon both by advocates and by opponents of the inclusion of a liquidity premium in the discount rate of insurance liabilities.

[8]End of 2008.

insurers argued that they should be allowed to include a liquidity premium in their discount rate.

The European Embedded Value (EEV) Principles [27, 65], which were issued in 2004 by the CFO Forum [24],[9] do not mention a liquidity premium. There is only a suggestion that the swap rate should be used as the risk free rate. It was only in 2008 that the liquidity premium was first mentioned, in the successor of the EEV Principles, the Market-Consistent Embedded Value (MCEV) principles. The first version of these principles was published in June 2008, and these principles still prescribed using swaps as the discount rate, with no illiquidity allowance,[10] reasoning that there was not enough consistency in the amount of liquidity premia [67].

Nevertheless, as the spreads widened and included a significant LP in 2008, by the start of 2009 many companies were effectively including liquidity premia in their 2008 year end MCEV disclosures. In the updated version of the MCEV principles as published in October 2009[11] by the CFO Forum, an entire principle is dedicated to the reference rate to use [28, 68], and Principle 14 now clearly mentiones that "the reference rate is a proxy for a risk-free rate, appropriate to [...] the liquidity of the liability cash flows", and further on explicitely imposes the use of a liquidity premium when the liabilities are not liquid.

End of 2009, Barrie + Hibbert (B+H) published a series of papers in favour of the use of liquidity premia in the discounting of insurance liability cash flows [64, 69, 70, 71, 72, 73].

Also end of 2009, CEIOPS[12] set up a Task Force consisting of the CFO Forum and the CRO Forum, the CEA,[13] AMICE,[14] Prof. Antoon Pelsser,[15] and the Groupe Consultatif[16] to discuss the inclusion of a liquidity premium for Solvency II valuation purposes [75]. Previously, similarly to the CFO Forum in the MCEV Principles, CEIOPS forbade the use of LP in its Consultation Paper 40 published in July 2009 [66]. In March 2010, the Task Force (TF) set up by CEIOPS published their "Report on the Liquidity Premium" [62]. This report, which is further discussed later on, was in favor of the use of a LP in the discount rate.

According to the TF, one of the big challenges was the estimation of the LP (on assets),[17] and

[9] See also chapter 2

[10] This was even explicitely forbidden in this version of the MCEV principles [66].

[11] This is the most recent version to date.

[12] CEIOPS stands for the Committee of European Insurance and Occupational Pensions Supervisors, which as of 2011 has been replaced by EIOPA, the European Insurance and Occupational Pensions Authority.

[13] The CEA was the Comitée Européen des Assurances, which as of March 2012 goes by the name of Insurance Europe (IE).

[14] AMICE is the Association of Mutual Insurers and Insurance Cooperatives in Europe.

[15] University of Maastricht.

[16] The Groupe Consultatif Actuariel Européen was established in 1978 to bring together the actuarial associations in the European Union to represent the actuarial profession in discussion with the European Union institutions on existing and proposed EU legislation which has an impact on the profession [74].

[17] The Report on the LP states "Where assets are illiquid, investors demand an additional premium as a reward [...]. The liquidity premium is only one component of the total spread between the yield of an asset and

the extent to which it should be applied, possibly only partially, to different types of liabilities. However, according to B+H [64], the fact that a LP is hard to measure does not preclude it from being applicable. According to the TF, the proportion of LP should be linked to the predictability of the liability cash flows.[18]

Based on a literature review [70], B+H propose different methods to estimate the LP for illiquid assets.[19] They mention that using different estimation approaches, empirical results are broadly consistent [71]. An extended discussion on different methods used to estimate LP on illiquid assets is however outside the scope of this master paper.

In July 2010, the Solvency II QIS5[20] exercise was organised by CEIOPS. The risk-free rates to be used included liquidity premia [2, 76].

As already mentioned in the previous chapter, the method used to derive the basis risk-free rate for Solvency was the one suggested by the CFO Forum and the CRO Forum QIS5 Technical Specifications Risk-free interest rates [60]. These specifiactions also prescribes the procedure to calculate the LP, taking into account the above mentioned research by B+H. This methodology has been used afterwards in MCEV publications up until now as well, see e.g. [17, 18, 19]. The formula for the LP is

$$\mathrm{LP} = 50\%(\varsigma - 40bp) \tag{4.1}$$

Where ς is the spread between corporate bond rates and (assumed risk-free) swap rates, which is determined as the difference between a government-corporate spread and a swap spread.[21] The evolution of the thus obtained LP over the last quarters is depicted in figure D.3 in appendix D.

According to the CFO Forum and the CRO Forum [60]:

> The corporate bond spread is considered to be comprised of three components: an allowance for the cost of default; a risk premium to compensate bond holders for bearing credit risk and a liquidity premium to compensate for the costs and associated uncertainty of trading illiquid bonds. Expected default costs over long horizons can be expected to be reasonably stable and we can interpret the deduction (y) as such an allowance for long-term expected losses. By setting the proportion (x) we split the remainder of the spread between the liquidity premium and the credit risk premium.

the liquid risk-free rate [...]. To determine the part of the spread attributable to liquidity risk, the challenge that has to be faced is the accurate breakdown of this spread into its components. [...]" [62].

[18] "The [...] majority of TF members consider that the illiquidity of an insurance liability measures thus the extent up to which its cash flows are predictable, i.e. are certain in amount and in timing. They recognize that this assessment is very complex [...]." [62].

[19] The three methods proposed are firstly an approach using CDS, secondly a structural model comparing illiquid risky bonds with liquid, but equally risky positions, and thirdly comparing risk-free illiquid bonds with a risk-free interest rate.

[20] QIS stands for Quantitative Impact Study. See also appendix B.

[21] As mentioned, a swap spread is taken as the difference between the swap rate and a government rate.

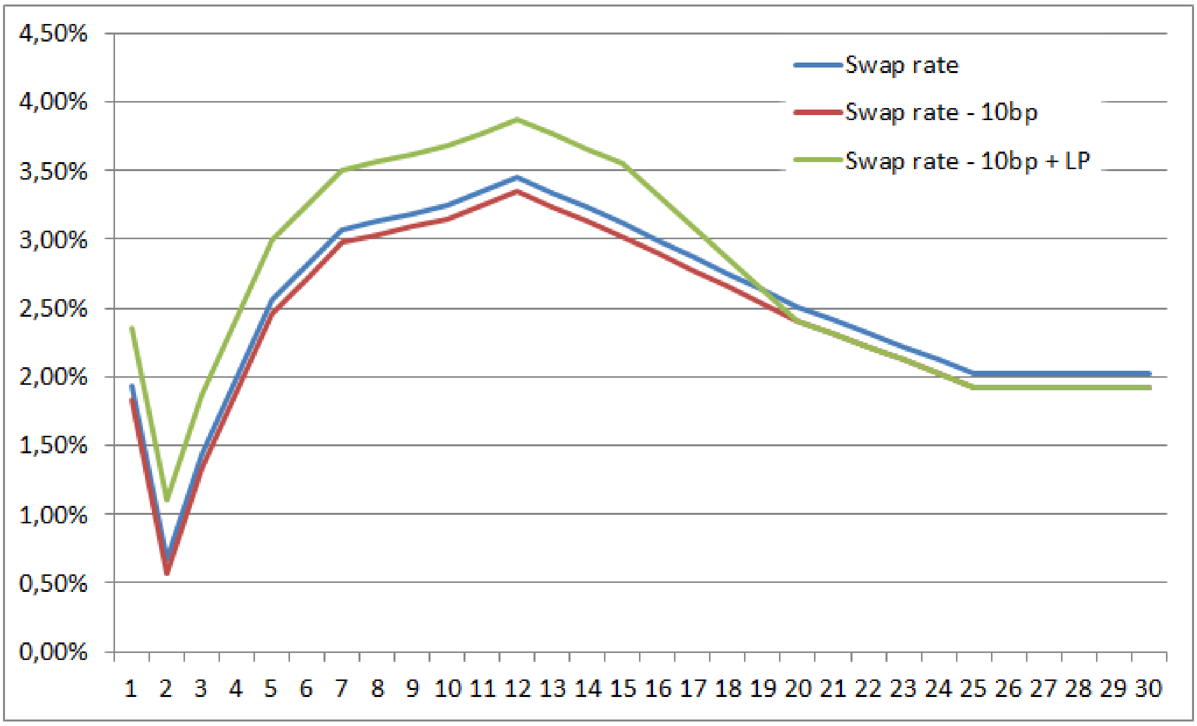

Figure 4.1: Constructing a risk-free term structure using the QIS5 method. Depicted rates are forward rates. First year rate is Euribor 12 months instead of 1 year euro swap rate.

ς is in practice estimated for a maturity which is between 3 and 5 years. The thus obtained asset LP is then transferred into a LP applicable to liabilities by using a percentage which is either 0%, 50%, 75% or 100%, based on the characteristics of the liabilities [2]. The thus obtained LP is then applied on the swap rate curve adjusted for credit spread[22] for 15 years and then linearly decreasing up until 20 years, where it is zero.[23]

An example of the adjustments is given in figure 4.1, using the QIS5 approach on 2011Q4 data. The swap curve is sourced from Bloomberg, the LP is, as mentioned in figure D.3, equal to 71bp. The asset-liability conversion proportion is taken to be 75%.

[22] I.e. minus 10bp.

[23] This was the euro method proposed for QIS5, but in the meantime, the CRO Forum stated that the LP should be applied up until the latest liquid point, which is currently estimated to be 30 years for euro swap rates [63]. Some insurers applied the former method in their 2011 MCEV reporting [19], others sticked with the QIS5 method [17, 18]. Because illiquidity in assets is mostly due to corporate bonds, which are not readily available for long terms, we believe the extension after 20 years is not a sound approach, based on the fact that a liquidity premium by which liabilities are discounted should be earned in the assets which are in reality in the corresponding hedging portfolio, as we will argue in the next chapter. We will not go into further detail about the practical application of the LP to the risk-free rate though. Another discussion point during QIS5 was whether to apply the LP on spot rates or on forward rates, where the latter is currently considered to be the most sound approach.

4.2 Premiums in the Draft Implementing Measures

A paper of April 2011 by the CRO Forum discussing the practical application of the LP [63] is one of the last papers that were written about the liquidity premium.

End October 2011, the European Commission shared the Draft Implementing Measures (DIM)[24] Solvency II [25] with the insurance industry. The most important articles for the purpose of this master paper can be found back in section B.4 in appendix B. In the DIM, there is not a word anymore about the liquidity premium. Instead, article 37 IR1 §1 states that the risk-free rate should include a matching premium (MP) and a counter-cyclical premium (CCP). The MP is discussed in section 4.2.1, the CCP in section 4.2.2. Both premiums have their origin in the LP.[25]

Note that there have been quite some documents published in response to the DIM. Because of the confidential nature of these document, and also because of their diverging contents, we only summarize the main headlines of these documents.

4.2.1 Matching premium

To be eligible for the use of a matching premium, a whole series of requirements to which insurance products must comply is put forward.[26] The asset portfolio should consist of bonds[27] only, there should be a ring-fenced structure,[28] there should be a nearly perfect cash flow matching between assets and liabilities,[29] there must be no surrender option or a surrender option with MVA, the assets should be of high quality . . . In fact, it is estimated that in Belgium there exist no insurance companies which will be allowed to use a MP in its current form. It will mostly be usable for annuity insurance in the UK or in Spain.

A lot of these requirements coincide with the restrictions we shall put forward in section 5.3.2 of the next chapter. Also, the way the matching premium is calculated, i.e. roughly by taking the difference between on the one hand the discount rate corresponding to a default-free version of the assets and on the other hand the basic risk-free rate, takes into account the characteristics of the assets backing the liabilities, another restriction we shall come back to in the next chapter.

Ever since the introduction of the stringent requirements to be eligible for the use of the MP, the industry has been arguing that the conditions mentioned needed to be relaxed. Because

[24] For some guidance on how Solvency II is organized, see appendix B.

[25] According to Keller [75], the liquidity premium lives on as the counter-cyclical premium, but from a theoretical point of view, the matching premium resembles more the liquidity premium.

[26] Some of which can be found back in section B.4 in appendix B.

[27] Or similar assets.

[28] A ring-fenced fund is "an arrangement where there is a barrier to the sharing of profits and losses arising from different parts of the undertakings business and/or own funds can only be used to cover losses on a defined portion of the entitys portfolio" [77].

[29] Where the asset cash flows are corrected for the default probability.

the LP was buried, it is needless to say that the response on the MP was huge, leading to a pile of documents, all arguing for an adaptation of the articles related to the MP. However, maybe because of the suddenly seriously reduced scope of applicability, and the therefore rather hasty reactions, there was perhaps too much confusion amongst different representatives. A lot of papers were citing a lot of changes that need to be made, because of a lot of different reasons. As of today, it seems that the insurance sector did not succeed in changing the DIM fundamentally concerning the MP.

Note further that the matching premium is calculated as the part of the spread which does not correspond to the default probability. The part corresponding to the default probability is termed the *fundamental spread*, while the remainder is the *non-fundamental spread*.[30] The non-fundamental spread thus contains not only a liquidity premium, but also part of the credit spread, as credit spreads are generally higher than default probabilities.

4.2.2 Counter-cyclical premium

Where the Matching Premium was designed to acknowledge the fact that the insurer might not be exposed to part of the spread, because the matching of his assets and liabilities is such that the risk is hedged, the counter-cyclical premium is designed to enable the industry to cope with extreme market conditions [78].[31]

Also for the counter-cyclical premium, quite some restrictions apply. Article 41 IR6 §1 of the DIM states that it will only be applicable in periods of stressed financial market, and that both the existence and the level of CCP is at EIOPA's discretion. The existence of financial stress is based on whether the spread exceeds the default probability, on whether a counter-cyclical measure is needed to counteract the fact that these assets will be sold, and on whether a fall is actually observed in the markets. Furthermore, article 41 IR6 §3 mentions that the CCP shall be calculated as a part of the spread between on the one hand the rate on a representative portfolio of bonds an insurer invests in, and on the other hand the basic risk-free rate. This part should not be due to any risk.[32]

Again, the insurance sector has been arguing to change the articles related to the CCP. The main discussion point is the fact that insurers want to have a transparant and fully documented calculation description of the CCP, including some sort of formula which will indicate the triggering of the CCP, and thus making an EIOPA intervention obsolete.

There have been quite some proposals on how the CCP should be calculated, none of which

[30] Although we are not particularly in favour of this terminology, we will use it anyway.

[31] The use of a CCP and the use of a MP are mutually exclusive, which is not too surprising, see DIM art. 41 IR6 §5.

[32] The latter statement is quite strange, since a spread is mostly due to some risk.

are in line with article 41 IR6 §3. In early papers,[33] some argued that the CCP should be calculated as consisting of a LP[34] *and* a government spread premium (GSP), *and* an additional discretionary component. The GSP is calculated by taking the difference between the *ECB AAA and other* government yield curve[35] and the risk-free rate at a certain maturity.[36] More recently, the industry argues that the CCP should only contain a maximum of the GSP and the LP, using the following formula to determine the GSP:

$$\mathrm{GSP} = y_{\mathrm{govt}}(\tau) - (r_{\mathrm{swap}}(\tau) - 10\mathrm{bp}) \tag{4.2}$$

where $r_{\mathrm{swap}}(\tau)$ is the euro swap rate, and where $y_{\mathrm{govt}}(\tau)$ denotes the government yield curve at maturity τ, and where the 10bp is the credit spread adjustment. The CCP is then added to the basic-risk free structure,[37] which is $r_{\mathrm{swap}}(t) - 10\mathrm{bp}$, thus obtaining:

$$r^{\mathrm{forward}}_{risk-free} = r^{\mathrm{forward}}_{\mathrm{swap}}(t) - 10\mathrm{bp} + \max\left(LP, y_{\mathrm{govt}}(\tau) - (r_{\mathrm{swap}}(\tau) - 10\mathrm{bp})\right) \tag{4.3}$$

It is not hard to see that this formula gets as close to cherry-picking as one can get without explicitely stating that the maximum between swap rates and government rates is taken.

Figure 4.2 compares the ten year euro swap rate with the mentioned ECB rate based on AAA and other government rates, while figure 4.3 compares the GSP calculated using equation 4.2 and compares it to the LP calculated using equation 4.1.

It is not clear yet to what extent the CCP concept is open for discussion, and to what extent the insurance sector can still come up with arguments to change the DIM. However, it is remarkable to see that quite some insurers already included a sensitivity estimation based on the GSP formula in their Embedded Value 2011 reporting, a practice which was recommended by the CFO Forum in December 2011 [17, 19, 81].

Given the magnitude of the GSP, which was around 186bp at 2011 year-end, it is needless to say that the impact of using the GSP is huge.

4.3 Conclusion

Since 2008, bond spreads have risen while risk-free rates have dropped. This leads to lower asset values, because of the spread increase, and lower liability values, if liabilities discounting

[33] See e.g. [79] or [80].

[34] Calculated using the above mentioned 50/40 approach.

[35] This is *not* the same yield curve as the one prescribed by the QIS4 Technical Specifications, since here, also lower rated government rates are included.

[36] E.g. at 10 year.

[37] Probably to the forward 1 year rate, see appendix A.

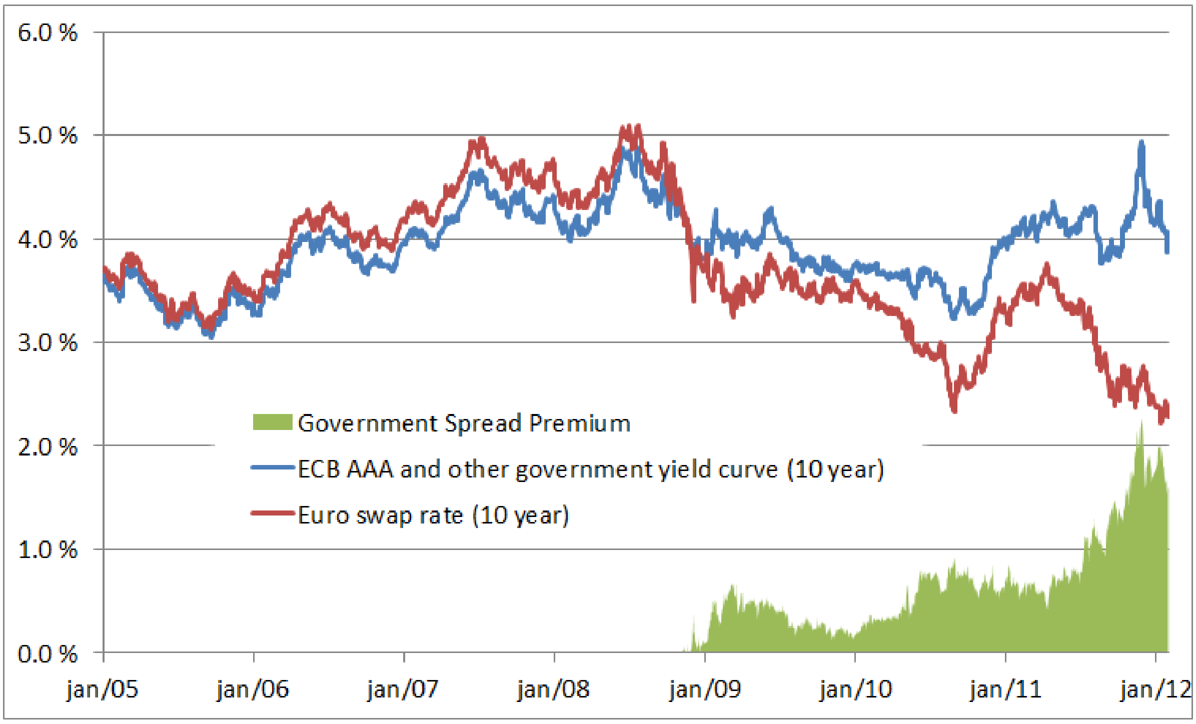

Figure 4.2: Evolution of the government spread premium as the difference of the 10 year euro swap rate and the 10 year ECB AAA and other government rate. Source: Bloomberg and ECB.

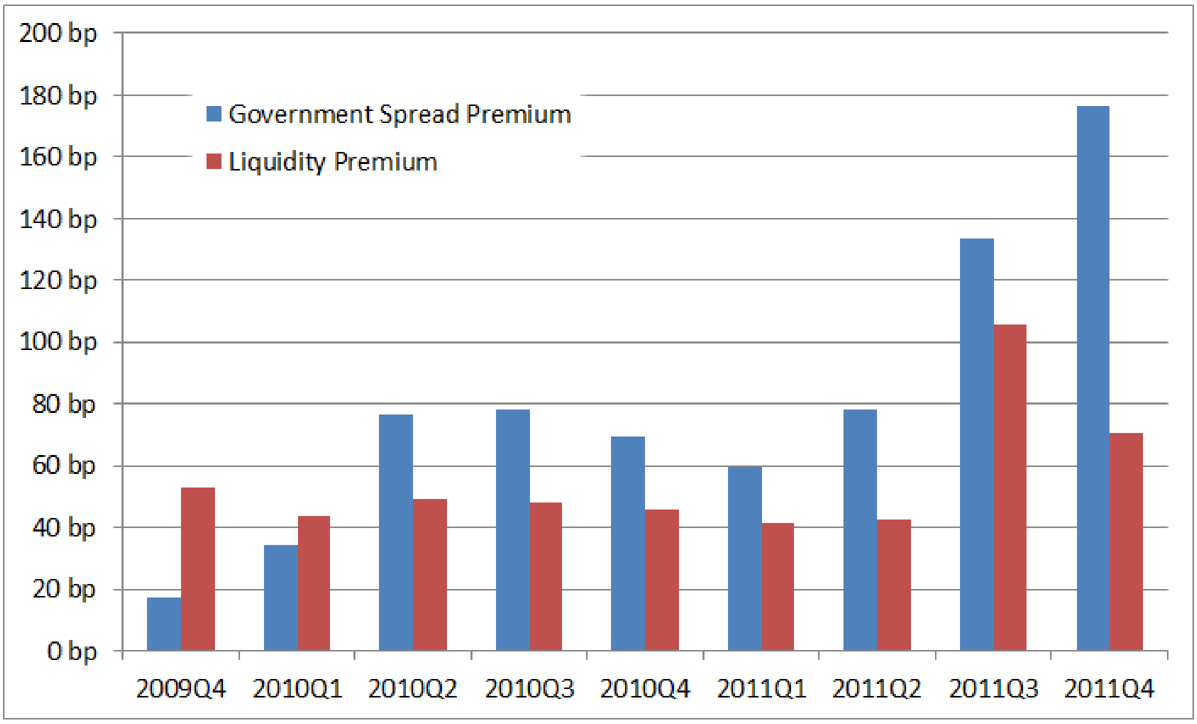

Figure 4.3: Evolution of the liquidity premium vs. the government spread premium.

is performed using interest rate models based on a purely risk-free rate.

However, part of the spread rise on the assets can be shown to be due to liquidity premia. Because insurers feel that their liabilities show liability characteristics, since 2008, they have been including liquidity premia on top of the risk-free rates in their discount rates used for MCEV purposes, and for their preparatory Solvency II exercises such as QIS5.

This inclusion has triggered quite some discussion on the jusitification of the inclusion of risk premia in the discount rate. Although we will postpone our full discussion on this justification to the next chapter, it must immediately be clear that the inclusion is not trivial. Indeed, one could argue that if insurers claim that they are not affected by part of the spread, they claim that arbitrage opportunities exist, and that they are able to exploit them, which makes the inclusion of a liquidity premium non-market-consistent. Although this goes against a basic finance intuition, we already make two remarks:

- Firstly, the arbitrage opportunity of investing in illiquid bonds while receiving an insurance premium which would be based on the risk-free discounted value of the benefits would indeed be an arbitrage opportunity. However, it is not necessarily sure that policyholders would be willing to pay the risk-free discounted value of the contract benefits, as the contract is not fully liquid.
- Secondly, arbitrage opportunities in the insurance sector do exist in the sense that margins can be taken. Using utility theory one can show that there is a difference between the price a policyholder is willing to pay for an insurance contract and the price an insurer is willing to accept for it.

Maybe partly because of discussions on the validity of the liquidity premium, the Draft Implementing Measures Solvency II replaced the liquidity premium by a matching premium and a counter-cyclical premium. The matching premium imposes very strict conditions and is in its current form only applicable for specific business in the UK and in Spain, while the counter-cyclical premium has the objective to deal with the pro-cyclical issues discussed in chapter 2.

Lastly, it is clear that because the insurance sector has been wounded by the current market environment, insurers are seeking ways to increase their value and lower their solvency *requirements*,[38] and in doing so might sometimes use threatening arguments.[39] This is both a natural and objectionable reflex. Nevertheless, it is not because some advocates of these adjustments might have ulterior motives, that what they are advocating is necessarily wrong.

[38]This does not necessarily mean that their actual *available* capital is therefore lower.
[39]See e.g. the extracts from the CEIOPS TF Report on the LP in the next chapter, where the italic parts clearly show market pressure on the regulator.

Chapter 5

Are derogations from the pure risk-free rate justified?

In the previous chapter it was mentioned that insurers feel they should be allowed to include a certain premium in their discount rate, and it was shown that both in MCEV reporting and in Solvency II, they will probably be allowed to do so, be it under certain restrictions.

Insurers often reason in a hold-to-maturity view, in which they hold their bonds until maturity and in which they don't care about price drops due to issues unrelated to the default probability, since they will hold the asset until maturity. This view is defendable, but some argue that it is not market-consistent.

In this chapter, we will try to find answers on whether the use of these premia is market-consistent or not, or whether there are other reasons why these premia might be justified. The discussion will mostly be based on liquidity premia (LP), since documentation on liquidity premia is far more established, easily available and reliable. It is not fully clear by looking at the Draft Implementing Measures what really is the full rationale behind the matching premium (MP), or behind the counter-cyclical premium (CCP). Therefore, the discussion on the MP and the CCP specific aspects will remain rather high level. Nevertheless, a lot of the conclusions drawn for LP are generally valid for any premium due to illiquidity which is added to the risk-free rate. Note that the analysis presented will not give answers on whether the full non-fundamental spread, of which only a part is due to illiquidity can be market-consistently included in the discount rate.

This chapter begins by discussing the main arguments used by advocates of the inclusion of a LP in the discount rate in section 5.1. Then, section 5.2 argues that one of the potential dangers with the use of a LP is the unreliability of its value. Thereafter, in section 5.3 it is investigated whether the inclusion of a LP leads to a non-market-consistent approach. It is argued that under certain restrictions, most notably the absence of a surrender option or a surrender option including a perfect Market Value Adjustment (MVA), market-consistent application of a premium due to illiquidity is possible. However, when a surrender option is present without a perfect MVA, the use of a LP might become questionable. Finally, section 5.4 concludes what can be learnt from this for the concepts of the MP and the CCP.

5.1 Why are insurance liabilities illiquid?

Insurers claim that their liabilities possess illiquidity characteristics, and that using a fully risk-free rate is thus incorrect to value their liabilities. We try to give an overview of why insurance liabilities could possibly be considered to be illiquid, and comment on the arguments as well.

5.1.1 Illiquidity because of limited possibility to cash in the liability

The CEIOPS Task Force[1] Report on the LP states:

> For the holder of an asset like a corporate bond, liquidity means the ability to sell or cash in this asset at any time [...]. Illiquidity occurs [...] where the asset is not readily saleable due to uncertainty about its value or due to the lack of a market in which it is regularly traded. [...] In the case of an asset represented by a claim on an insurance company, in many cases the *policyholder may be unable to sell this asset in the absence of a market or his ability to cash in the policy value may be limited by legal or contractual constraints or by financial penalties* [...]. Such insurance claims present clear illiquidity characteristics [...]

The TF thus argues that insurance liabilities are illiquid because the contract cannot be cashed in by policyholders, either because it can not be sold in the market[2] or the possiblitity to surrender is constrained. Often, if a surrender option exists, it includes a severe penalty, like a Market Value Adjustment (MVA) mechanism[3] The same reasoning is to be found back in e.g. a CRO Forum publication [63], where it is stated that "while the starting point for a market-consistent valuation should be observed market prices for equivalent financial instruments, these financial instruments are usually highly liquid (typically tradable daily), whereas the insurance liabilities they are intended to replicate are often not as liquid" [63], implicitly stating that insurance liabilities are not tradable daily and are therefore illiquid.

This argument advocates for an illiquidity characteristic which we feel does not necessarily influence the value of the insurance liability, if the link with the assets is not made. It does not matter *to the insurer* whether the liability is easily tradable or not, he has the obligations to pay the liability cash flows anyway. A non-insurance company emitting an illiquid corporate

[1] See previous chapter.
[2] Or at a serious discount, as in life settlements.
[3] A perfect MVA charges the difference between the market value and the book value as a penalty when surrender occurs. Other methods might include the actual sale of underlying assets [82].

bond can still try to buy back the illiquid bond in the market, receiving an illiquidity discount, but as mentioned on page 39, insurers tend to have a hold-to-maturity view, and they will in practice not buy back the liability. The insurer might have the possibility to propose the policyholder an alternative payment scheme, but it is questionable whether the client would be willing to settle for the discounted value at a rate including a LP.

5.1.2 The illiquid asset replication argument

A more elaborate reasoning on why liquidity premia should be included, which starts from the argument mentioned in section 5.1.1, but takes it one step further, is the following:[4]

Life insurance liabilities are, at least up to a certain extent, predictable. This predictability depends among others[5] on the right of the policy holder to surrender his policy. Suppose now that the insurer wants to hedge his position, and he does this by setting up a replicating portfolio. In doing so, the insurer has the opportunity to invest either in liquid bonds, or in equivalent bonds which are illiquid. Since the insurer (because of the predictability of the liabilities) will not need to sell the asset before it runs off, the insurer will choose the illiquid bond. Since insurance liabilities are valued using a replicating portfolio approach, and the best matching replicating portfolio consists of illiquid assets, the risk-free curve to discount the liabilities should thus be adjusted for the illiquidity of the assets in the replicating portfolio.

The CEIOPS TF hints at this argument as well, stating [62]:

> It is common practise to cover illiquid insurance liabilities with highly predictable cash flows with similarly potentially illiquid assets with corresponding maturities *the alternative to such an approach would be an increase in the price of products for consumers.* The appearance of an important liquidity premium implicitly contained in the valuation of these assets created a shortfall in the balance sheet of the concerned companies and the insurance industry claims this shortfall to be artificial insofar that in case of an efficient hedge against credit risk, the revenues of the assets, both regular and at maturity, were not at risk and were sufficient to match the cash outflows of the insurance contracts. The introduction of a liquidity premium in the valuation of insurance liabilities aims at eliminating this valuation mismatch and *avoiding the situation that such investments no longer become an option for companies with a detrimental impact on both consumers and financial markets.*

Note that one of the big *possible* problems with allowing for a liquidity premium without the correct restrictions is demonstrated exactly by this reasoning. If the insurer invests in risk-free *liquid* bonds, while discounting his liabilities using a rate which includes a liquidity premium,

[4]Variations of this argument stating that "A liability is liquid if the liability cash flows are not reasonably predictable" exist, see e.g. [68], but they are misinterpretations of arguments that make more sense.

[5]It also depends on an insurance component (mortality) and a financial component, which insurers might argue they can hedge. Some [39, 1] are not to convinced of this.

he will soon run in to problems.[6] It is clear that if insurers want to use a LP for their liabilities, that they can only do this up to the extent that they earn the liability premium on their assets. This will also be concluded in section 5.3.2, using a mathematical example.

5.1.3 A false proof by example

An often cited proof by example constructed by advocates of the LP, and which could support both arguments mentioned above, is that two identical policies, except that one of the policies includes a surrender option, while the other one does not, cannot have the same value.

Although this is true, this example is not necessarily a proof of the necessity of the inclusion of a premium in the discount rate.

The surrender option obviously has a cost for the insurer, but this option should be valued the same way as other options offered to the policyholder, such as profit sharing. One could use a stochastic valuation process, were the surrender rate possibly depends on the stochastic scenario. Of course, reasonable assumptions need to be set for this option,[7] but this is outside the scope of this master paper.

As an example, consider a non-puttable bond,[8] for which their exists a liquid market to trade the bond. It makes no sense to say that this bond has illiquidity characteristics, just because the principal can not be demanded directly from the bond issuer. This bond is as liquid as a hypothetical puttable bond for which their exists a liquid market to trade it. Although it is clear that the pricing of these two liquid bonds can be done without the use of a LP, the prices of the two bonds are obviously not the same. The embedded option in the puttable bond can be valued using a rather basic pricing model. This shows that the difference in value due to surrender options does not imply the necessity of a LP in the discount rate.

However, consider a puttable bond, being illiquid in the sense that the bond is hard to trade. This is a lot like an insurance contract, which has a surrender option. If this bond were to be valued using a stochastic valuation process, the exercise of the put option would depend on the difference between the uncertain illiquid value of the contract in the market, and the redeemable principal. Whether a LP could be useful in finding this illiquid value of the contract in the market, is not fully clear, see also section 5.3.3. So although the value of a surrender option is not a proof of the necessity of a liquidity premium, the two might be interrelated.

[6] Because if the thus calculated value of assets and liabilities is equal at inception, then necessarily the rate of return will be lower on the risk-free asset then the guaranteed rate on the illiquid liabilities.

[7] One has to take into account that, unlike an often made assumption in finance, the surrender option is far from always executed as soon as it is financially/economically sound to do so and thus consideration has to be given to historic surrender rates in combination with expert judgement [83].

[8] I.e. a regular bond, without the option to demand early repayment of the principal.

5.2 A first problem: Estimating the liquidity premium

Liquidity premia have always been a controversial subject. Already in December 2009, Crugnola-Humbert and Gallacher of Deloitte Zürich,[9] expressed their concerns on both theoretical and practical aspects of the illiquidity premium in a very interesting paper [66]. One of the issues mentioned by Crugnola and Gallacher is that industry convergence was needed regarding the determination of the liquidity premium.[10]

This issue is larger than it appears at first sight. First of all, it is important to note that estimating liquidity premia for assets is already hard. As mentioned, various methods have been suggested and are used, but the problem with most of these methods is that they work by difference. Calculating a total spread is easy when a risk-free referentiel has been found. The credit spread part can be estimated via methods using credit default swaps, or by comparing very liquid, but otherwise equally risky rates with very liquid risk-free rates or other methods [71]. But what then remains is the liquidity premium, plus a remainder. It's easy to say that if we take an illiquid risky investment combined with a CDS, that the remainder is due to illiquidity, but there is no proof. For example, an investor investing in an illiquid asset might also require a risk premium because of the price uncertainty, since the correct price for an illiquid instrument is often unknown [66].[11] Secondly, even if the liquidity premium on assets is determined and agreed upon, the question remains how this should be correctly transfered to a liquidity premium on liabilities? As already mentioned, advocates of the LP acknowledged that the LP on assets is only partially applicable to the liabilities, depending on their predictability. But using a 0%, 50%, 75% and 100% bucket approach, as required by the Solvency II QIS5 Technical Specifications, seems to be rather approximative.[12] Thirdly, convergence in the industry was needed. Despite the theoretical difficulties this seemed not so much of a problem, as both for the Solvency II QIS5 exercise, and for the MCEV publications since 2010, the industry used the 50-40 approach mentioned in section 4.1.

A similar concern on the magnitude estimation of the LP was raised in March 2010, at the same time when the CEIOPS Task Force published their report, by Keller,[13] who mentions that a problem with the argument is that by transferring the problem of valuing insurance liabilities to a setting in which illiquid corporate bonds are used, is that prices for illiquid bonds are inherently unreliable, and that market prices could deteriorate easily [39].

[9] In Switzerland, Solvency II will not be applicable, and as already mentioned, instead the Swiss Solvency Test is applicable, which uses Swiss government bond rates as the risk-free rate, without the use of an additional LP.

[10] This issue was already mentioned in section 4.1 in the previous chapter.

[11] Of course, this price uncertainty premium is also due to the illiquidity, but an insurer cannot argue that it shouldn't consider this part of the spread when pursuing a hold-to-maturity strategy.

[12] Although the approach from a modelling point of view is quite transparant.

[13] Member of the Swiss Actuarial Association, and, like Crugnola-Humbert and Gallacher, also working for Deloitte Zürich.

5.3 Is a LP market-consistent?

As already mentioned, some feel that the use of a LP is not market-consistent.[14]

In March 2011, the Dutch National Bank invited academics for a Workshop on Liquidity Premium in SII: Conceptual and Measurement Issues. The academics unanimously rejected the validity of the use of the LP for insurance liabilities [75], and published an interesting paper after the Workshop [84].

One of these academics, Wüthrich, went on to publish a destructive report in April 2011 on the use of the LP [1]. Wüthrich has on the one hand a couple of non-mathematical counter-arguments, which have mostly been cited above,[15] and, more interestingly, a mathematical counter-argument, which will be discussed in section 5.3.1. This section aims to give an easy-to-comprehend version of the original argument.

Section 5.3.2 discusses a possible flaw in the reasoning made by Wüthrich, and turns the example provided in an argument in favour of the LP.

Because the example of Wüthrich is based on quite some restrictions, most notably the absence of a surrender option, we try to include a surrender option by valuing an illiquid puttable bond in section 5.3.3. We conclude that in this case, the use of a LP can be contested.

All three sections are also useful for the matching premium and for the counter-cyclical premium, see further, which beginning of 2011 were still only in an embryonic phase.

5.3.1 Wüthrichs argument against the liquidity premium

5.3.1.1 Sketching the example

The argument in the original paper by Wüthrich is based on pricing using a real-world analysis and using deflators, but it can be simplified without needing the rather technical and complicated formulas in the paper as follows.[16]

[14] E.g. Keller [39] feels a LP has no place in a market-consistent view, although it is a correct argument in a hold-to-maturity view. Keller goes on to propose an alternative, which is to use the illiquid rate to discount CFs and include the extra risk in the Solvency II risk margin. Theoretically, this should have no influence, as the difference with the approach where the basis risk-free rate is used is passed on to the risk-margin. The risk margin is then calculated using a proxy cost of capital method. This way, the risk margin relies on long-term averages, as it is determined as a the present value of all future SCR times a cost-of-capital rate, and the method reduces volatility. This idea seems cumbersome, as the correct inclusion in the risk margin is probably very hard. Furthermore, this idea does not increase the already limited transparancy on the components of the risk margin, and at the same time it blurs the value of the liabilities (excl. risk margin), making it unreliable and meaningless.

[15] The counter-arguments are the fact that predictability counterdicts insurability and that it is unmeasurable, that a LP is hard to measure, that predictability is hard to relate to illiquidity, and finally that market prices of illiquid instruments are erratic.

[16] Note that this section (5.3.1.1) and the next two sections (5.3.1.2 and 5.3.1.3) are practically entirely based on the paper by Wüthrich, but the argument is greatly simplified here, without losing the essence.

The analysis uses discrete time steps of a year.[17] Denote the price of a risk-free zero-coupon bond price with maturity T at time t by $P(t,T)$. Denoting the price of a risky zero-coupon bond by $B(t,T)$, it is logic[18] that:

$$B(t,T) \leq (1-p)^{T-t} P(t,T) \Gamma_t \tag{5.1}$$

where p is the yearly default probability, and where Γ_t is a binary stochastic variable that indicates whether the bond has already defaulted. The inequality can be defended by the fact that the investor requires an additional return because of the default risk, and because of the fact that the risky bond is potentially illiquid.

Based on this equation, we make the following assumption:

$$B(t,T) = (1-s)^{T-t}(1-p)^{T-t} P(t,T) \Gamma_t \tag{5.2}$$

with $0 \leq s \leq 1$. We thus assume that the non-default-probability part in the price of the spread behaves geometrically with respect to time, which is defendable.[19]

Let us now price a basic insurance liability portfolio, consisting of a cash flow of size 1 at time $T = 2$, without any surrender option. This is a fully predictable liability. Denote the assets at time t by $\mathcal{A}_t$, and the liabilities by $\mathcal{L}_t$.

Every year the insurer will have to make sure he still has enough assets to back the liability. If not, he is insolvent, and he will have to reinject capital. If he has more assets, we assume the value is directly upstreamed to the shareholder. This will lead to a *consumption stream* $(\mathcal{C}_0, \mathcal{C}_1, \mathcal{C}_2)$, where $\mathcal{C}_t = \mathcal{A}_t - \mathcal{L}_t$. We further assume that the policyholder is charged the fair premium π, which is $\pi = P(0,2)$.[20] The asset investments are done in risky bonds with market value $B(t,T)$.

5.3.1.2 Valuation using a risk-free rate

Suppose now that the valuation is done using a risk-free curve without allowance for a liquidity premium. Then, $\mathcal{L}_0 = P(0,2) = \pi = \mathcal{A}_0$, thus $\mathcal{C}_0 = 0$. Because of equation 5.2, it follows that

$$\mathcal{A}_0 = (1-p)^{-2}(1-s)^{-2} B(0,2) \tag{5.3}$$

[17] This doesn't matter, but it eases the equations.

[18] Assuming investors are either risk-neutral of risk-averse, which is reasonable.

[19] Although the fact that interest swap rates tend to be lower than interbank rates for the same maturity, see e.g. figure 3.2 indicates that this assumption is probably not entirely correct in practice.

[20] Note that $P(0,2)$ is determined using a risk-free rate without a liquidity premium.

At $t = 1$, we get:[21]

$$\mathcal{C}_1 = \mathcal{A}_1^- - \mathcal{L}_1 = [(1-p)^{-1}(1-s)^{-1}\Gamma_1 - 1]P(1,2) \tag{5.4}$$

Since this consumption can be positive or negative, depending on whether the bond defaults, the company will need to inject capital. We assume that new capital is again invested in risky bonds, and that part of the bond is sold to upstream the value to the shareholder if the consumption is positive. We thus get $\mathcal{A}_1 = \mathcal{L}_1 = P(1,2)$, which leads to:

$$\mathcal{A}_1 = (1-p)^{-1}(1-s)^{-1}B^{(1)}(1,2) \tag{5.5}$$

where $B^{(1)}(1,2)$ is the price of a risky bond, conditional on the event that it has not defaulted in the first period. Equivalently to equation 5.4, this leads to:

$$\mathcal{C}_2 = \mathcal{A}_2^- - \mathcal{L}_2 = [(1-p)^{-1}(1-s)^{-1}\Gamma_2^{(1)} - 1] \tag{5.6}$$

where $\Gamma_2^{(1)}$ is the default process $\Gamma_2 | (\Gamma_1 = 1)$.

As a consequence of no-arbitrage, it follows that the price of the consumption stream at time 0 is equal to zero. If not, the premium charged to the policyholder was not the fair one. Furthermore, the expected gains generated by the consumption stream are:

$$\mathrm{E}_{\mathbb{P}}[\mathcal{C}_0] = 0, \quad \mathrm{E}_{\mathbb{P}}[\mathcal{C}_1] = \frac{s}{1-s}\mathrm{E}_{\mathbb{P}}[P(1,2)] > 0, \quad \mathrm{E}_{\mathbb{P}}[\mathcal{C}_2] = \frac{s}{1-s} > 0 \tag{5.7}$$

This rightly shows that, if a premium $\pi = P(0,2)$ is charged, investing in risky bonds is on average[22] rewarded by a positive gain.

5.3.1.3 Valuation using a risk-free rate plus liquidity premium

Now let's assume that a LP is used in the discounting process. To facilitate things it is *assumed that the entire part of the spread on top of the pure credit probability is due to illiquidity of the risky bond.* The liability as a function of time is then

$$\mathcal{L}_0 = (1-s)^2 P(0,2), \quad \mathcal{L}_1 = (1-s)P(1,2), \quad \mathcal{L}_2 = 1 \tag{5.8}$$

[21] By using equation 5.3 and replacing $B(0,2)$ by $B(1,2)$.
[22] Using a real-world probability measure $\mathbb{P}$.

We still assume that the policyholder is charged with the premium $P(0,2)$.[23] Because we again require $\mathcal{A}_0 = \mathcal{L}_0$, and because of equation 5.2, we get

$$\mathcal{C}_0 = \pi - \mathcal{A}_0 = s(2-s)P(0,2) > 0 \tag{5.9}$$

Using the same kind of analysis as above, it is easy to see that

$$\mathcal{C}_1 = [(1-p)^{-1}\Gamma_1 - 1](1-s)P(1,2) \tag{5.10}$$
$$\mathcal{C}_2 = [(1-p)^{-1}\Gamma_2^{(1)} - 1] \tag{5.11}$$

The no-arbitrage price at time $t = 0$ is again 0. The expected consumptions under the real world-probability measure $\mathbb{P}$ are now given by:

$$\mathrm{E}_{\mathbb{P}}[\mathcal{C}_0](2s - s^2)P(0,2) > 0, \quad \text{and} \quad \mathrm{E}_{\mathbb{P}}[\mathcal{C}_1] = \mathrm{E}_{\mathbb{P}}[\mathcal{C}_2] = 0 \tag{5.12}$$

Because the gain at time $t = 0$ is strictly positive, the *value* of the remaining consumption stream $(\mathcal{C}_1, \mathcal{C}_2)$ is necessarily negative.

According to Wüthrich, if an insurer really uses a LP to discount liabilities, too much value is upstreamed to the shareholder in a too early phase, leaving the insurer with a portfolio which has negative value afterwards. The immediate release, in contrast to a release over time, gives rise to a weakened financial strength afterwards.

5.3.2 Using the example to conclude in favour of the liquidity premium

It is not trivial that an insurer who includes a LP in his discount rate would charge a premium of $P(0,2)$ to the policyholder. Instead, if the pricing department considers the rate including the LP as the risk-free rate,[24] the policyholder will be charged with $\pi' = (1-s)^2 = \mathcal{A}_0 = \mathcal{L}_0$.

In words this means that the insurer reasons in a world were the spread which is not due to default probability is filtered away. In such a world there is thus no compensation for the risk-bearing, as insurers in such a world are risk-neutral. We would call this view a hold-to-maturity view, because under no circumstance, the insurer is forced to sell his bonds before arriving at maturity, meaning he is not exposed to the illiquidity risk.

Applying the same exercise as in the previous sections, we then obtain

[23] This is not as trivial as Wüthrich makes it look. We will therefore also perform the exercise for another premium value in the next section.

[24] This does happen in practice.

$$\begin{aligned}
\mathcal{C}_0 &= 0 && (5.13)\\
\mathcal{C}_1 &= [(1-p)^{-1}\Gamma_1 - 1](1-s)P(1,2) && (5.14)\\
\mathcal{C}_2 &= [(1-p)^{-1}\Gamma_2^{(1)} - 1] && (5.15)
\end{aligned}$$

Taking the real-world expectation values leads to:

$$\mathrm{E}_{\mathbb{P}}[\mathcal{C}_0] = \mathrm{E}_{\mathbb{P}}[\mathcal{C}_1] = \mathrm{E}_{\mathbb{P}}[\mathcal{C}_2] = 0 \tag{5.16}$$

Remember that in the previous section (5.3.1) we mentioned that the no-arbitrage value of the consumption stream was 0, because the premium charged was the fair one, being $P(0,2)$. Indeed, if we reason that the insurance product the policyholder buys is a risk-free cash flow of 1 at time $t=2$, he will be willing to pay the price of a risk-free zero-coupon bond paying 1 at time $t=2$, being $P(0,2)$, since the bond and the insurance contract are the same.

However, advocates of the LP could argue that the no-arbitrage value of the consumption stream mentioned in *this* section is zero, based on the following argument.

The insurance contract might be less easy to trade then the risk-free zero-coupon bond, the latter being fully liquid by definition of risk-freeness. Because the insurance contract is less liquid, the policyholders will be probably willing to pay less then $P(0,2)$ because the product they are buying contains illiquidity risk. If the investor takes into account the illiquidity he would rather pay $(1-s')^2P(0,2)$ where s' is some parameter taking into account the illiquidity of the insurance policy. Because the insurance sector, in some countries maybe even more than in others, is a competitive one, insurers might offer products at a premium which is indeed lower than $P(0,2)$. If one then assumes that $s=s'$, then the no-arbitrage price of the contract is indeed 0, since the investor/policyholder paid the same price he would pay for an equivalent product.

The assumption $s=s'$ is perhaps not obvious, since the illiquidity characteristic of the insurance contract is probably not entirely the same as the one of the illiquid risky bond B. This is the problem of the transfer of the LP from the asset side to the liability side. Note however that from an insurer point of view, he should never let $s'>s$, as this would lead to negative values at policy inception. This means that an insurer can only offer a product with a liability discount to the extent that he can obtain the liability discount on illiquid bonds which he uses as his assets to back the liability.

We could thus conclude that in this example, given that $s' \leq s$, the insurer should possibly be allowed the use of a LP.

Note however, that the example discussed is based on quite some assumptions.

1. Firstly, in the example, there is no surrender option whatsoever. The conclusions drawn would not change if we included a surrender option which includes a perfect Market Value Adjustment (MVA) mechanism. However, if there is a surrender option which does not include a perfect MVA, the insurer could get into trouble as the insurer can then not reason the same way as done in this section. Moreover, the contract discussed is not even an insurance contract, since there is no insurance risk.
2. Secondly, it is assumed that the insurer invests in risky illiquid bonds. Note that if he does not do so, and he invests in risk-free bonds instead, the insurer will run into problems if he uses a LP in the discounting of his liabilities.[25]
3. Thirdly, in this example, we assumed that the spread can be split up into a default probability spread and a liquidity premium. However, in practice, credit spreads are larger than default probabilities, and there might be a remainder which is due to other risk sources. The example does not lead to any conclusion about whether it is allowed to include the full part of the spread which is not the default probability in the discount rate. It only says that in this example, it could be justified to include the *liquidity* premium.
4. Furthermore, we assumed that p and s were known. In practice however, the price of illiquid bonds is by definition unreliable, which makes the estimation of components of their spread unreliable as well. Because of the unreliability, an insurer willing to use a liquidity premium in his discount rate, should thus make prudent assumptions on its magnitude.
5. Lastly, we assumed a perfect cash-flow match between assets and liabilities.

Extending these restrictions is not trivial, especially if one includes surrender options, which enter some sort of liquidity into the product. This is attempted in the next section.

5.3.3 Are a liquidity premium and a surrender option reconcilable?

While in the previous section, we showed that the use of a liquidity premium in the discount rate might be reconcilable with a market-consistent approach if there is no surrender option, in this section, we try to find out whether the use of a liquidity premium is still justified if the insurance policies does include a surrender option. We try to do so by approximating an insurance contract which includes such an option, by an illiquid puttable bond. Note however that, as already mentioned, in insurance contracts, surrender options in insurance are not necessarily exercised if it is favourable to do so. Furthermore, we take a policyholder or bond

[25] This is the $s' \leq s$ argument mentioned.

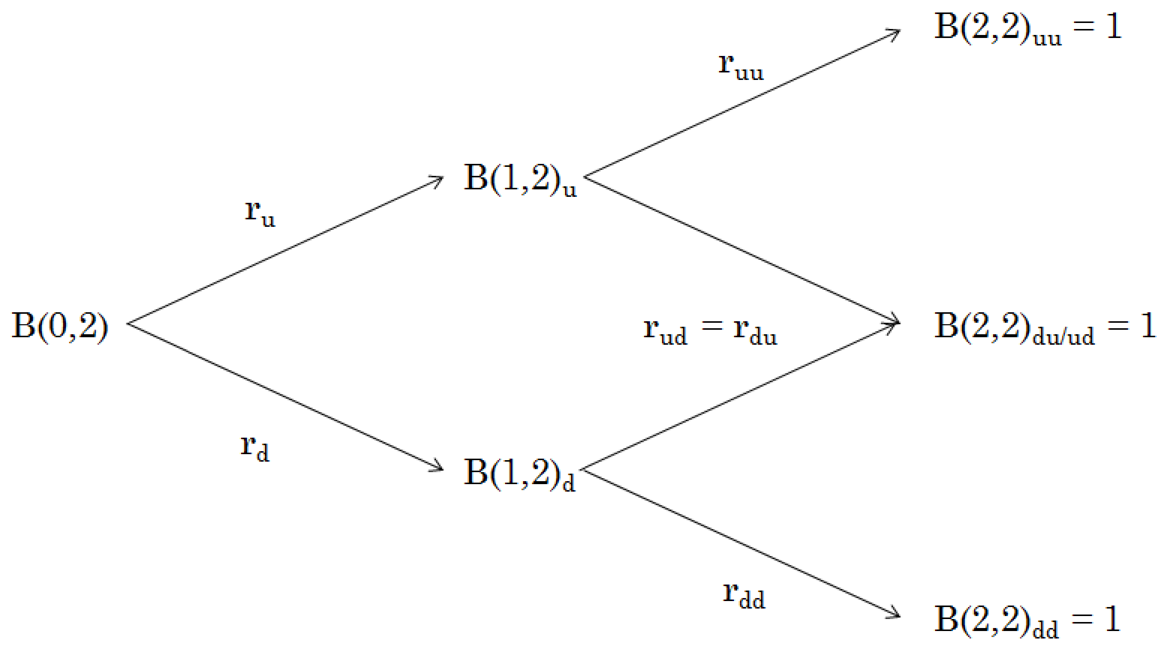

Figure 5.1: A two-step binomial tree pricing model

holder, point of view, regarding the contract as an asset. Our reasoning is thus not necessarily valid for the insurer, who has the contract as a liability on his balance sheet.

We consider a very basic two step interest rate binomial tree for our pricing, to price default-risk-free bonds with a pay-off of 1 at maturity. The model is presented in figure 5.1. We denote the price at time t of a bond with maturity at time T by $B(t,T)$. Furthermore, we denote the stochastic variable of the risk-free interest rate in the first year by R_1, which has possible outcomes r_u and r_d, while the stochastic variable of the risk-free interest rate in the second year is denoted by R_2, with possible outcomes r_{uu}, $r_{du} = r_{ud}$, and r_{dd}. Note that the risk-neutral probability measure can be chosen by requiring that the interest rate model is arbitrage free. We shall not go into numerical details and stick to a formulaic approach, but the outcome should make sure that $\mathrm{E}_{\mathbb{Q}}\left[\frac{1}{R_1}\right] = \frac{1}{r_1}$, and $\mathrm{E}_{\mathbb{Q}}\left[\frac{1}{R_2}\right] = \frac{1}{r_2}$, where r_1 and r_2 are the current forward 1 year rates implied in the current risk-free term structure.

The pricing of a liquid, non-puttable bond is done much like the binomial tree pricing of the call option in chapter 2. This leads to the following set of equations allowing for a price at $t = 0$:

$$B(1,2)_u = \mathrm{E}_{\mathbb{Q}}\left[\frac{1}{1+R_2}|R_1 = r_u\right] \tag{5.17}$$

$$B(1,2)_d = \mathrm{E}_{\mathbb{Q}}\left[\frac{1}{1+R_2}|R_1 = r_d\right] \tag{5.18}$$

$$B(0,2) = \mathrm{E}_{\mathbb{Q}}\left[\frac{B(1,2)}{1+R_1}\right] \tag{5.19}$$

Now we consider an illiquid, non-puttable bond. The illiquidity is modelled by assuming that there is an illiquidity risk leading to a potential loss of part of its *fundamental* value[26]. If we model this loss as a stochastic parameter S, and we consider S and R to be independent, we obtain:

$$B(1,2)_u = \mathrm{E}_{\mathbb{Q}}\left[\frac{1-S}{1+R_2}\middle| R_1 = r_u\right] = \mathrm{E}_{\mathbb{Q}}\left[1-S\right]\mathrm{E}_{\mathbb{Q}}\left[\frac{1}{1+R_2}|R_1 = r_u\right] = \mathrm{E}_{\mathbb{Q}}\left[\frac{1-\mathrm{E}_{\mathbb{Q}}\left[S\right]}{1+R_2}|R_1 = r_u\right] \tag{5.20}$$

$$B(1,2)_d = \mathrm{E}_{\mathbb{Q}}\left[\frac{1-S}{1+R_2}\middle| R_1 = r_d\right] = \mathrm{E}_{\mathbb{Q}}\left[1-S\right]\mathrm{E}_{\mathbb{Q}}\left[\frac{1}{1+R_2}|R_1 = r_d\right] = \mathrm{E}_{\mathbb{Q}}\left[\frac{1-\mathrm{E}_{\mathbb{Q}}\left[S\right]}{1+R_2}|R_1 = r_d\right] \tag{5.21}$$

$$B(0,2) = \mathrm{E}_{\mathbb{Q}}\left[B(1,2)\frac{1-S}{1+R_1}\right] = \mathrm{E}_{\mathbb{Q}}\left[1-S\right]\mathrm{E}_{\mathbb{Q}}\left[\frac{B(1,2)}{1+R_1}\right] = \mathrm{E}_{\mathbb{Q}}\left[B(1,2)\frac{1-\mathrm{E}_{\mathbb{Q}}\left[S\right]}{1+R_1}\right] \tag{5.22}$$

It is obvious that an equivalent pricing model be obtained by disregarding the possible illiquidity loss, but by using a discount rate which includes a liquidity premium. This liquidity premium, to first order, is equal to $\mathrm{E}_{\mathbb{Q}}\left[S\right]$.

Now suppose that the bond contains an embedded put option, which allows the bondholder to sell the bond for a certain value K at time $t = 1$. Then in every step, we consider the maximum of the illiquid value in the market, and the value for which the bond can be sold to the issuer:

$$B(1,2)_u = \mathrm{E}_{\mathbb{Q}}\left[\max\left(\frac{1-S}{1+R_2}, K_u\right)|R_1 = r_u\right] \tag{5.23}$$

$$B(1,2)_d = \mathrm{E}_{\mathbb{Q}}\left[\max\left(\frac{1-S}{1+R_2}, K_d\right)|R_1 = r_d\right] \tag{5.24}$$

$$B(0,2) = \mathrm{E}_{\mathbb{Q}}\left[B(1,2)\frac{1-S}{1+R_1}\right] \tag{5.25}$$

Note that now, because of the max-function, it is not generally possible to separate the expected values into an expected value depending on S and another depending on R_2, unless for some specific K. Note e.g. that if $K = \frac{1-S}{1+R_2}$, we obtain the same value as for an illiquid bond without put option. This expression for K might be compared to a MVA in an insurance contract.

Nevertheless, in this basic example, we conclude that for an illiquid bond, which contains an embedded put option, it is not generally possible to use an equivalent pricing model where the

[26] By fundamental value, we mean the expected discounted next time step value, under the risk-neutral probability measure. Although the use of a loss in every time step might seem strange, it is rather consistent with the notion of a liquidity premium in a risk-free rate.

liquidity loss is ignored, but the illiquidity is priced by the use of a liquidity premium in the discount rate.

Because the model presented is only a very basic one, and because we only consider the inclusion of a surrender option, further research on the extension of the restrictions metioned in the previous section is urgently needed.

5.4 Lessons learnt for the matching premium and the counter-cyclical premium

In section 5.1 we discussed why insurers feel they should include a LP in their discount rate. In section 5.2 we showed that one of the potential dangers with the use of a LP is the inherent unreliability of its value. Adding a premium to the risk-free rate is a controversial subject, and many, particularly in the academic world, feel that it is a non-market-consistent approach. Nevertheless, by considering an example used by an opponent of the inclusion of a LP in discount rates, in section 5.3 we argued that under certain restrictions, one being the absence of a surrender option, or a surrender option with a perfect MVA,[27] the inclusion of a LP in the discount rate can still lead to a market-consistent approach. A preliminary study of the validity of using a LP when a surrender option *is* included in the contract presented in section 5.3.3, indicates that the use of a LP might not be a valid approach. In any case, further research on the extension of the restrictions is urgently needed.

What can be learnt from this for the MP and CCP concepts?

In any case, the issue of the uncertainty in estimation of premia remains. The fact that estimating LP is hard, is true by definition, as price uncertainty is inherently associated with illiquidity. Although B+H are right to say that this does not prevent it from being applicable, caution is required. Therefore, if premia are estimated in the context of Solvency II, as article 76§4 requires, this should be done in a reliable and prudent way. Furthermore it is not because the industry quickly converges on a methodology to use, as it did for the LP, that this methodology therefore gives a reliable and prudent estimation.

The inclusion of a LP is justified either by using the argument that liabilities are illiquid directly because the policyholder has a limited possibility of cashing his provisions or by arguing that the liabilities can be replicated and hedged by using illiquid assets, again because the policyholder has a limited possibility of cashing his provisions. As mentioned, one of the biggest problems

[27] At least in Belgium, insurers now widely apply so-called MVAs when a client surrender, since they are allowed to do so by the Royal Decree concerning Life Insurance Article 30§2. Note however that the method allowed in there is by far not a perfect MVA, unless all assets are invested in Belgian Government bonds. For group insurances, when a big transfer is made of reserves, the Belgian law does allow for a perfect MVA, since in that case, the insurer can apply a cost model which is defined in the initial agreement.

with the LP was that it was not required that the liabilites are also in practice hedged by illiquid assets. If an insurer buys liquid risk-free bonds to hedge the liability cash flows he has, and he then discounts the liabilities using a discount rate including a LP, then he will have a cash flow matching problem.

The matching premium as currently defined in the DIM is applicable under strict requirements only. Based on the results from section 5.3, we might conclude that the use of a matching premium is indeed justified. Furthermore, by imposing requirements on the assets held, the matching premium also addresses the cash flow matching problem.

However, there remain three key issues to be further investigated.

1. First of all, although the inclusion of a liquidity premium might be justified, it is not clear whether the full non-fundamental spread is eligible for inclusion. The example in section 5.3 only deals with the liquidity premium. This should be further investigated.
2. Secondly, even if it could be justified to apply the entire non-fundamental spread, we feel there is not enough prudency in the estimation of the matching premium, as default probabilities are estimated on a very long time scale, see Article 42ter§3. It is clear that a default probability might change over time, due to different economic conditions.[28]
3. Thirdly, it should be noted that it is not because we believe that under the stringent conditions imposed by the DIM, the use of a premium in the discount rate might be justified, that there should be no premium whatsoever as soon as one of the conditions is not or only partially satisfied. The analysis in section 5.3.3 indicates that the use of a LP might not be correct when an illiquid asset is puttable, but it does not argue that the use of a risk-free rate is correct either.[29] We therefore stress again that further research on relaxing these conditions is needed.

As for the CCP, we believe that it indeed might have its desired counter-cyclical effect. However, we feel that there are two issues with the CCP.

The first issue is the lack of a theoretical justification for the CCP, since there is no proof of the validity of a premium when the strict conditions of the MP are relaxed. If the one and only purpose of the counter-cyclical premium is to introduce counter-cyclicality, then we believe it should be done away with. If instead a sound theoretical reasoning can be developped, as is currently the case for the MP given its strict requirements, then it could be justified. Although counter-cyclical measures might be necessary, the introduction of a premium on top of the basic risk-free rate without a theoretical justification leads to meaningless, and intransparant numbers, which is a disadvantage still outweighing the advantage of having less volatile numbers.[30] If

[28]This holds even for assets of the same rating, see e.g. hull2005bond.
[29]Unless of course the illiquidity risk is modelled explicitly in another way.
[30]Insurers often talk about counter-cyclical measures as measures to reduce “artificial volatility”.

counter-cyclical measures are deemed necessary anyway, other possibilities need to be looked for.

Secondly, the current proposal of using a GSP as CCP, which is mentioned in 4.2.2 does not have a sound theoretical basis and it should therefore not be used. If a theoretical justification for the inclusion of another premium then the MP is found, than ideas of calculating the CCP should be developped starting from this theory, not from wishful thinking.

Chapter 6

Conclusion

This master paper discusses which interest rate should be used in a market-consistent valuation model of insurance liabilities. Such models are required for Market Consistent Embedded Value (MCEV) reporting, and for Solvency II purposes.

In chapter 2 we started by explaining some basic finance models, discussing the pricing of a European call option using a binomial tree, thereby introducing the notions of real-word and risk-neutral probability measures, and indicating how more complex models such as a Black-Scholes equity model, and a Hull-White interest rate model function. All of these models require the risk-free interest term structure as an input parameter. Then, we argued how these finance models can be used to obtain a value of an insurance policies portfolio, which is consistent with observed market prices. Because as mentioned, such an approach is required in Solvency II, and because the impact on calculated value is enormous, especially for long-term business, the choice of the interest rate to use is currently a big subject of debate. To conclude the chapter, the pros and cons of the use of finance techniques to value insurance balance sheets were discussed. There are clear advantages to the use of such market-consistent techniques, such as increased risk-sensitivity and increased transparancy, because “smart accounting” is a lot harder. Nevertheless, there are also possible issues with applying finance techniques to an insurance balance sheet. Firstly a theoretical issue, which is illiquidity, secondly, there is a possible unintended economic consequence, which is pro-cyclicality, and thirdly, the fact that smart accounting is harder does not mean it is impossible, it mostly makes it harder to spot.

In chapter 3, we try to find risk-free rates in the market that can be used in insurance valuation models. We assume that real interest rates in the market are composed of a risk-free component, and a spread, consisting of a credit spread, a liquidity premium, and a remainder. As a first candidate, we then discuss the government rates, which have the advantage of being liquid and which are available for rather long maturities. Nevertheless, it is clear that European government bonds are currently far from risk-free. Using only AAA rated European government bonds is not an option, because it is envisagable that at some point in the future, no such bonds might be available anymore. Furthermore, government bonds might possess a negative convenience spread, making them unsuitable for use as risk-free rates. A second candidate discussed are interbank rates, such as Euribor, which are again highly liquid, and which have been used by banks traditionally as risk-free rates. The problem with interbank rates though is that they are mostly not available for maturities higher than one year. It is clear that for use in insurance valuation models, longer maturities are definitely needed. Furthermore, also in interbank rates, since end of 2007, there is a considerable amount of credit spread. The third canditate considered are interest swap rates. They have the advantages of being highly liquid, of having lower credit spread than interbank rates for the same maturities, and they are available for rather long maturities. Swap rates, adjusted for credit spread, are indeed the rates currently prescribed as the basic risk-free interest rates to use in both MCEV and Solvency II. Although we agree with the use of swap rates, we show that there is considerable evidence indicating that the current credit spread in swap rates could be up to ten times higher then the currently estimated value of 10bp.

Since 2008, spreads have increased, while risk-free rates have dropped. This leads to lower bond market values, because overall rates increased, and higher modelled insurance liability values, because of the low risk-free rates. Moreover, since 2008, liquidity premia have become clearly visible in spreads on corporate bonds. Insurers feel that because of the illiquidity of their liabilities, and because they feel they are not exposed to certain other risks, they should be allowed the use of liquidity premia in the discount rate used for the discounting of their liabilities.

In chapter 4, we show that such derogations are allowed in the MCEV Principles, and in the Solvency II QIS5 Technical Specifications in the form of a liquidity premium. In the Draft Implementing Measures Solvency II, the application of a matching premium or a counter-cyclical premium is allowed, but the requirements are much more restrictive.

In chapter 5, we try to find out whether the use of these premiums is theoretically justifiable, because using them has always been a controversial approach. For example, Wüthrich believes that "it is not the task of the actuary to introduce fancy arguments [...] that circumvents the current regulation". In a paper [1], he uses a mathematical example to show that the use of a liquidity premium is non-market consistent. However, by changing a questionable assumption made in this paper, we feel that the conclusion should rather have been that in this particular example, the use of a liquidity premium *does* lead to a market-consistent approach. The example presented is based on quite some restrictions though, so that we are not yet able to draw conclusions about whether a liquidity is justified in more general cases as well. One of the restrictions of the example by Wüthrich is that their is no early surrender option. A preliminary analysis made here indicates that when a surrender option is included, the use of a liquidity premium might be invalid. Nevertheless, further research on in what conditions the use of a liquidity premium can be justified, is urgently needed.

Because the matching premium is much more restrictive in its applicability, and because the restrictions show a lot of similarity with the restrictions under which we showed the use of a liquidity premium might be justified, we believe the matching premium might be justified as well. However, we see two key issues. Firstly, it is not clear whether these restrictions are the largest set under which a premium is justified. As already mentioned, further research should bring more clarity on this. Secondly, the estimation of the magnitude of the matching premium is currently based on an estimation method for that part of the spread which does not correspond to an average default probability over a long time period. This non-fundamental spread as it is called, contains both a liquidity premium and another component. It is not clear whether the full non-fundamental spread should be added to the risk-free rate, even under the strict restrictions of the matching premium. On top of it, the use of an average default probability over time is felt to be a very bad, and unprudent approximation for the current default probability.

Because of the possible pro-cyclical consequences of using a market-consistent approach, we believe in the necessity of counter-cyclical measures. In any case, even if market consistent accounting would be only con-cyclical instead of pro-cyclical, the introduction of counter-cyclical measures might be a good "bonus pater familias" idea. We even believe that theory, and a blind belief in market-consistency, sometimes has to be put aside. As mentioned in the introduction, consideration should also be given to social arguments. As the ultimate aim of Solvency II is providing better protection to the policyholders, a theoretical method having pro-cyclical consequences should be adapted. Nevertheless, we believe that the use of a counter-cyclical premium, which is an adaptation of the risk-free rate as counter-cyclical measure, is a bad idea. Although we believe that a counter-cyclical premium might indeed have a counter-cyclical effect, we feel that if there is no theoretical justification for applying a premium, no premium should be applied. If it were to be applied without a theoretical justification, this would lead to meaningless, and intransparant numbers, on which no sound risk managment can be based. Without theoretical justification, we feel applying a counter-cyclical premium would just be a smart actuaries' trick. Therefore, other, more transparant counter-cyclical measures than a counter-cyclical premium should be looked for.

Appendices

Appendix A

Zero-coupon rates, par yields, and forward rates

This appendix explains the differences between interest rates quoted in spot format (zero-coupon rates), par yield format and forward format.[1]

We shall base our explanation on the following example. We suppose that in the market, there exist three truly risk free zero-coupon bonds. An overview of these zero-coupon bonds, their Time to Maturity (TTM), their face value and their price is given in Table A.1.

	ZC Bond 1	ZC Bond 2	ZC Bond 3
TTM	1 year	2 years	3 years
face value	10	10	10
price	9.9010	9.6117	9.1514

Table A.1: Example: Three zero-coupon bonds, their time to maturity and their price.

It is not hard to see that the Yield To Maturity (YTM) of these three bonds are respectively 1%, 2%, and 3%. To obtain these YTM, we search the rate at which the single cash flows[2] has to be discounted to obtain the price. So for example, the YTM of zero-coupon bond 2 is given by $\sqrt{\frac{9.6117}{10}} - 1 = 2\%$. The YTMs can thus be used to discount a single CF at a certain time. We call these rates zero-coupon rates or spot rates. The spot rate at a given maturity thus corresponds to the interest rate that would be earned on a zero-coupon bond [6], or equivalently the spot rate at a given maturity can be used to obtain the present value of a single CF at that maturity by a straightforward discounting. In general, the present value (PV) of a series of

[1] An analogue explanation, which starts from a continuous rate can be found back in most finance textbook, such as Hull [6].

[2] All three bonds have no coupons, so there is only the cash flow of the principal.

yearly cash flows can then be calculated as:

$$\mathrm{PV}\left(\mathrm{CF}\right) = \sum_i \frac{\mathrm{CF}_i}{\left(1 + r_i^{\mathrm{spot}}\right)^i} \tag{A.1}$$

Or, defining discount factors $P(0,i) = \frac{1}{\left(1+r_i^{\mathrm{spot}}\right)^i}$:

$$\mathrm{PV}\left(\mathrm{CF}\right) = \sum_i \mathrm{CF}_i P(0,i) \tag{A.2}$$

Suppose now that we have a fourth and a fifth bond, both with face value 1000, with TTM 2 years and with yearly coupon payments. Bond 4 has a coupon rate of 2%, while bond 5 has a coupon rate of 1.99%.

Let's now determine the price of these two bonds, given the information we have in table A.1 and using equation A.1. Bond 4 will provide us with a cash flow of 20 after one year and with a cash flow of 1020 after two years. This is equivalent to owning 2 of the above mentioned zero-coupon bonds with a TTM of one year and 102 of the zero-coupon bonds with a TTM of two years. The price of bond 4 should thus be $2 \times 9.9010 + 102 \times 9.1514 = 100.0194$. Notice that this price is higher than the face value, although the bond provides us with a coupon rate equal to the spot rate at its maturity. Applying the same arbitrage freeness reasoning for bond 5 shows us that its price should be 100, which is equal to its face or par value. This is why the coupon rate of bond 5 is called the par yield, i.e. the par yield at a given maturity is the coupon rate of a coupon bearing bond with that maturity and which sells at par. The par yield is equal to the YTM of both bonds 4 and 5.

Because of the definition of par yields, the following needs to hold:

$$1 = P(0,j) + r_j^{\mathrm{par}} \sum_{i=1}^{j} P(0,i) \tag{A.3}$$

and therefore:

$$r_j^{\mathrm{par}} = \frac{1 - P(0,j)}{\sum_{i=1}^{j} P(0,i)} \tag{A.4}$$

The rates of (long-term) government (or corporate) bonds are generally quoted in par yield format. Also the interest swap rate is quoted in par format. Therefore, one sometimes uses the term swap rates to describe rates in par yield format, a habit which should be avoided to avoid misunderstandings.

Equation A.4 can be used to compute par yields starting from spot rates, or inversely, to compute spot rates from par yields. This is e.g. done in figure 3.4, where spot rates are derived from the swap rate which is quoted in par yield format.

Although it might seem reasonable that bond rates are quoted in par format, it is perhaps less obvious that swap rates also quoted in par format. However, if one considers a swap transaction as the exchange of two at par bonds, one paying a floating rate coupon bond, the other paying a fixed rate[3] coupon bond, this should be no surprise.

The last format in which an interest rate is often expressed is the discrete forward rate or the continuous (forward) rate. Suppose that an investor emits 101 zero-coupon bonds of type 1, for which he receives 1000. He will thus need to pay back 1010 in one year. Furthermore, he invests the 1000 he received for the selling of these bonds in zero-coupon bonds of type 2, which will pay him 1040.40 in two years. From a point of view of the investor, he thus has two remaining cash flows, i.e. -1010 in 1 year and 1040.40 in two years. This means that the investor will have earned an interest rate of a little more than 3% during the second year. This is a 1 year forward 1 year rate in year 2, meaning that it is the implied 1 year rate in the second year. Equivalently, the 2 year forward 1 year rate is the implied 1 year rate during the third year. In general the x year forward y year rate is the y year rate during the period starting with the $x+1$-th year and ending at the end of the $x+y$-th year. We use the term forward (1 year) rates in general to describe x year forward 1 year rates.

Note that these rates are *not* the rates that will be effectively the future rates, since future interest rates are a stochastic process. One could assume though that they represent some kind of market expectation of future rates. Furthermore, forward rates are future rates that can be locked in today, leading to a gain or loss in the future, depending on whether the real interest rate ends up below or above the rate you locked in previously.

The relationship between forward 1 year rates r_i^{forward} and zero-coupon rates is:

$$\frac{1}{\left(1+r_i^{\text{spot}}\right)^i} = \frac{1}{\prod_{j=1}^{i}\left(1+r_j^{\text{forward}}\right)} \tag{A.5}$$

In finance theory, one often works with continuous (forward) rates. $r(t)$ is then a continuous time-dependent variable, and discounting over a period from v back to τ is then performed

[3] I.e. the par yield.

using a discount factor which is equal to:

$$P(\tau, \upsilon) = \int_{\upsilon}^{\tau} \mathrm{e}^{r(t)t} \mathrm{d}t \tag{A.6}$$

Because $\mathrm{e}^r \approx 1 + r$ for r small, continous rates tend to be close to 1 year rates, although even when interest rates are low, there is a difference of some basis points.

Appendix B

A brief intro to Solvency II

This appendix aims to give an introduction to Solvency II, the solvency regulation which will be applied shortly in the European Union. The info presented below should provide the reader with enough background to comprehend the other sections dealing with Solvency II in this master paper. Firstly, section B.1 gives a brief overview of the solvency regulations applicable in Europe. This section is based quite extensively on [24]. Secondly, section B.2 explains the main ideas used in Solvency II. Thirdly, section B.3 gives an idea of the organisation of the Solvency II project, and tries to situate the current position in the planning. Finally, section B.4 contains some parts of the Draft Implementing Measures Solvency II dealing with the matching premium and the counter-cyclical premium, as these parts are probably not readily available to every reader of this master paper.

B.1 History of European solvency regulations

Because of the direct impact of an insolvency of an insurer on the financial situation of individuals, the insurance sector is of fundamental economical and social importance. Therefore, a variety of regulations for the insurance sectors have existed since the mid-1800s [85]. Nowadays, there is a general consensus that insurance supervision is necessary to minimize an insolvency scenario [86].

Since the 1970s, the European Commission (EC) regulates the supervision on life insurance activities in the First Directive Life Insurance.[1] Since the First Directive Non-Life Insurance,[2] which was issued in 1973, there were already requirements on the solvency margin for insurers with non-life activities [87]. The Third Directive Life Insurance[3] of 1992, brought some minor changes, but mainly focused on industry convergence [85]. These directives allow a wide choice

[1] Directive 79/267/EEC.
[2] Directive 73/239/EEC.
[3] Directive 92/96/EEC.

in valuation techniques. On top of this, the member states were allowed to apply more stringent rules, which lead to significant differences in practice between the member states, but also between different insurers [86].

In 1997 the EC started with an evaluation of the existing solvency regulation. This revision was initiated by the Müller Report of 1997, which revealed weaknesses of the existing regulation and put emphasis on the need for a more risk-sensitive approach [86]. On the one hand, this lead the EC to change the outdated thresholds in the existing regulations, an intiative which is termed Solvency I, and which was endorsed in 2004 [85]. On the other hand, this lead to the project Solvency II, which will be further discussed in the next session.

In the meantime, it is worth noting that in Switzerland,[4] the Swiss Federal Office of Private Insurance developed the Swiss Solvency Test (SST) in close cooperation with the insurance industry and academic representatives [88]. The project began in 2003 and was field tested in 2004 and 2005. In 2006, the new framework became applicable for large insurers and, since the beginning of 2008 it is in effect for all insurers.

B.2 Main ideas

Solvency II has much wider ambitions than Solvency I. The goal is, via more risk-sensitive capital requirements and governance, to arrive at a better protection of the policyholder [89]. The improved risk-sensitivity in the capital requirements is to be attained by the use of an economic, market-consistent approach in risk valuation [2, 90].

In a study by KPMG in 2002 [86], commissioned by the EC, it is mentioned that a market valuation or a market-consistent approach would increase transparancy in the true financial situation of an insurer. This market-consistent approach is also the one retained by the SST [31], which, as mentioned, is currently the solvency regulation applicable in Switzerland.

The advice of KPMG was followed, and Solvency II introduced two quantitative capital requirements, the Minimum Capital Requirement (MCR) and the Solvency Capital Requirement (SCR). In short, the MCR is the level which may never be breached, while the SCR is the target level, and is risk-based.[5] The MCR is based on easy factor-based rules,[6] while the determination of the SCR is based either on a standard model, or on an internal model. The SCR should correspond to the capital required to remain solvent with a probability of 99.5% in the coming year. In the standard model, the SCR is calculated in different risk-modules. In quite some of these modules, the SCR is calculated as the impact of a shock-scenario on

[4]Switzerland is not part of the European Union.

[5]The SCR shouldn't be breached either, but consequences are less severe.

[6]Like the rules in Solvency I.

the *Net Asset Value* (NAV) [2], i.e. the difference between the fair value of assets and the fair value of liabilities.

These different ΔNAV per submodule are then combined via a correlation matrix to obtain one single value for the SCR.[7]

But what is a “fair value”? Article 75 of the Solvency II Directive[8] states that

> (a) assets shall be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arms length transaction
> (b) liabilities shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arms length transaction.

Furthermore, Article 76 mentions:

> 2. The value of technical provisions shall correspond to the current amount insurance and reinsurance undertakings would have to pay if they were to transfer their insurance and reinsurance obligations immediately to another insurance or reinsurance undertaking.
> 3. The calculation of technical provisions shall make use of and be consistent with information provided by the financial markets and generally available data on underwriting risks (*market consistency*).
> 4. Technical provisions shall be calculated in a *prudent, reliable and objective* manner.

Article 77 mentions:

> 1. The value of technical provisions shall be equal to the sum of a best estimate and a risk margin as set out in paragraphs 2 and 3.
> 2. The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure. [...]
> 3. The risk margin shall be such as to ensure that the value of the technical provisions is equivalent to the amount that insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations.

In short, assets should generally be valued using a mark-to-market approach, while liabilities should be valued using a mark-to-model approach. As the model should be market-consistent,[9] a risk-free rate is a very important input (see chapter 2). Note that Article 77 mentions that Solvency II, like the Market Consistent Embedded Value (MCEV) Principles, acknowledges that there needs to be an adjustment for the fact that the model can not give an exact value of the liabilities, since part of the liabilities can not be hedged. Therefore, the market is incomplete and an extra adjustment is needed. In Solvency II this is called the Risk Margin, in MCEV this is called the Cost of Residual Non-Hedgeable Risk.

[7] The real calculation is slightly more complicated in fact, but this is unimportant for the purpose of this master paper.
[8] Directive 2009/128/EC.
[9] See Article 76.

Furthermore, it is important to stress that Solvency II is based on a transfer view (see Article 76 and 77), and that the three key requirements for valuing liabilites are prudence, objectivity and reliability.

B.3 Organisation and timeline

The Solvency II project is organized in two phases [91]. In the first phase, from May 2001 to April 2003, fundamental arrangements were specified, a general framework was defined, and several studies were ordered by the European Commission. In the second phase (beginning December 2003), these fundamentals are being developed into specific rules and guidelines, with frequent opportunities for input from member states and relevant stakeholders. Not only is input possible, but it is specifically sought by the EU as part of the regulatory development process.

The new rules are being created following the Lamfalussy procedure. This implies that Solvency II is introduced in different steps. The Framework Directive[10], which was adopted in 2009, is restricted to the basic principles, which has made the discussions in the Council and in the European Parliament considerably easier. This has often been seen as an advantage, but the disadvantage is that much of the new regime still needs to be written in the Implementing Measures (level 2) and in supervisory guidance (level 3) [92].

The Lamfalussy process is based on extensive consultation of all stakeholders. According to an article van Hulle published in 2011 [92],[11] this consultation is burdensome for the Commission as well as for stakeholders, but it is definitely rewarding and it leads to better law making.

Part of this consultation is done in the form of Quantitative Impact Studies (QIS), which are executed by the industry. The QIS serve to test the models and to enhance their calibration [93]. The fifth and currently latest QIS is QIS5, which was held end of 2010. The QIS5 technical specifications give a reliable insight in the view at that time of the regulator on quite some technical aspects.

In the same article as mentioned above, van Hulle mentions that the deadline for the level 2 Implementing Measures is 31 October 2012, although the European Commission has announced that it will propose to move that date to 31 December 2012, which is a technical adaptation aiming at making the new solvency regime coincide with the calendar year. Solvency II would thus have to be applied for the first time from 1 January 2013. This proposal is part of the Omnibus II directive, which still has to be voted by the European Parliament. At the moment, it seems rather unlikely that this deadline will be attained however. According to EIOPA,

[10] Directive 2009/138/EC.

[11] Karel van Hulle is the Head of the Insurance and Pensions Unit at the European Commission.

"EIOPA has adopted a working assumption for the timeline of Solvency II, taking as a basis the entry into force of Solvency II from 1 January 2014" [94].

In October 2011, the Draft level 2 Implementing Measures were shared with the insurance sector. One of the issues which is still under discussion in these Implementing Measures[12] is the risk-free rate, which is of course partly why this master paper deals with this subject.

B.4 Matching premium and counter-cyclical premium

This section contains some parts of the articles in the Draft Implementing Measures Solvency II [25] which deal with the matching premium and the counter-cyclical premium.

Article 37 IR1 §1 states:

> The rates of the relevant risk-free interest rate term structure to calculate the best estimate with respect to insurance or reinsurance obligations, as referred to in Article 77(2) of Directive 2009/138/EC, shall be calculated as the sum of:
> (a) the rates of a basic risk-free interest rate term structure;
> (b) where applicable, a counter-cyclical premium
> (c) where applicable, a matching premium.
> [...]

On the counter-cyclical premium, article 41 IR6 states:

> 1. The counter-cyclical premium shall be zero for all maturities except during periods of stressed financial markets as determined by EIOPA. EIOPA shall determine the existence of a period of stressed financial markets if the following conditions are met:
>   (a) a material part of the spread between the rates of credit risk-free liquid assets and the rates of assets in the representative portfolios of assets can be demonstrably attributed to the illiquidity of those assets or a credit spread that exceeds the credit risk of the issuer;
>   (b) it is demonstrated that the illiquidity or excess credit spread of the assets in the representative portfolios of assets, referred to in point (a), is more likely than not to result in undertakings selling a large part of those assets unless a counter-cyclical premium is taken into account in discounting technical provisions
>   (c) there is a fall in financial markets which is unforeseen, sharp and steep.
>
>   [...]
> 3. For each currency, the counter-cyclical premium shall be calculated in a transparent, prudent, reliable and objective manner as a portion of the spread between the interest rate that could be earned from assets included in a representative portfolio of assets that insurance and reinsurance undertakings are invested in and the rates of the basic risk-free interest rate term structure. The portion shall not be attributable to a realistic assessment of expected losses or unexpected credit risk on the assets. The portion shall not be attributable to any other risk. [...]

[12] Whilst the level 2 Implementing Measures are being prepared and discussed, EIOPA is also working on its level 3 supervisory guidance.

5. [...] The counter-cyclical premium shall not be applied to insurance and reinsurance obligations where the relevant risk-free interest rate term structure to calculate the best estimate for those obligations includes a matching premium. [...]

Article 42bis deals with the application of the matching premium. Paragraph 1 of this article states:

The risk-free rate interest rates to calculate the best estimate of a portfolio of insurance obligations shall include a matching premium, determined in accordance with Article 42ter, provided that the following conditions relating to the insurance obligations and the assets covering them are met:

(a) the insurance undertaking has assigned a portfolio of assets, consisting of bonds and other assets with similar cash-flow characteristics, to cover the best estimate of the portfolio of insurance obligations and intends to maintain this assignment over the lifetime of the obligations, except for the purpose of maintaining the replication of cash-flows between assets and liabilities where the expected cash-flows have materially changed such as the default of a bond;

(b) the portfolio of insurance obligations and the assigned portfolio of assets are ring-fenced, managed and organised separately from the other activities of the insurance undertakings, without any possibility of transfer;

(c) the future cash-flows of the assigned portfolio of assets replicate each of the expected future cash-flows of the portfolio of insurance obligations in the same currency; any mismatch shall not give rise to risks which are material in relation to the risks inherent in the insurance business to which a matching premium is applied;

(d) the insurance contracts underlying the portfolio of insurance obligations do not give rise to future premium payments;

(e) the only underwriting risks connected to the portfolio of insurance obligations are longevity risk, expense and revision risk; the contracts underlying the insurance obligations include no options for the policy holder or only a surrender option where the surrender value does not exceed the value of the assets, valued in accordance with Article 75 of Directive 2009/138/EC, covering the insurance obligations at the time the surrender option is exercised; [...]

(h) no assets of the assigned portfolio of assets shall have a credit quality which has been assigned to credit quality step 4 or worse [...]

Article 42ter deals with the calculation of the matching premium:

1. The matching premium referred to in Article 42bis(1) shall be equal to the difference of the following:
   (a) the annual effective rate, calculated as the single discount rate that, where applied to the cash-flows of the portfolio insurance obligations, results in a value that is equal to the value in accordance with Article 75 of Directive 2009/138/EC of the portfolio of assigned assets;
   (b) the annual effective rate, calculated as the single discount rate that, where applied to the cash-flows of the portfolio insurance obligations, results in a value that is equal to the value of the best estimate of the portfolio of insurance obligations where the time value is taken into account using the basic risk-free rate term structure. The calculation in

point (a) shall only consider the assigned assets whose de-risked cash-flows are required to replicate the cash-flows of the portfolio of insurance obligations, excluding any assets in excess of that. For this purpose, a 'de-risked cash-flow' of an asset means the expected cash-flow of the asset where the expected default of the asset is taken into account in accordance with a probability of default that corresponds to the fundamental spread and a loss-given-default [...]

2. The fundamental spread, referred to in paragraph 1, of a specific asset shall be equal to the sum of the following:
   (a) the credit spread corresponding to the probability of default of the asset;
   (b) a spread corresponding to the expected loss resulting from downgrading of the asset;
3. [...] The probability of default referred to in paragraph 2(a) should be based on long-term default statistics that are relevant for the asset in relation to its duration, credit quality step and asset class. [...]

Appendix C

The impact of risk-free rate adjustments on value

This appendix aims to show that adding premiums to a risk-free rate does not necessarily increase the calculated value of an insurance contract.

Of course, in most cases there is no doubt that increasing the risk-free rate used to discount the liabilities will increase value. There is hardly any impact on the asset valuation, since they are mostly valued at their market price. The impact on the liabilities, which are mostly positive in total, and thus represent an outflow for the insurer, is that they will be discounted more, hence lowering the present value of the liabilities and increasing the Net Asset Value.

However, as the contract boundary discussion[1] in Solvency II seems to tend towards a less restrictive and thus wider approach, more future premiums will be taken into account, together with their associated policyholder benefits. It is not so difficult to see that this extra part might well have a sensitivity which is not like the one usually assumed, but rather the other way around.

Indeed, consider (a part of) a contract, for which premiums still have to be paid in the future, but for which no assets are bought yet, and which, due to the contract boundary widening would now lie within the boundary. There is no interest rate guarantee or profit sharing heritage from the past,[2] so at the moment of valuation, the insurer can still decide upon which guarantees to give. Because of this, the value of this (part of the) contract will likely be positive. Since however there are no assets at valuation time, the liabilities should be negative. Applying the same reasoning as above leads to the conclusion that it might be that the value drops when the risk-free rate is adjusted upwards.

[1]As already mentioned in the main text, see [24] for a discussion.

[2]If there was, the premiums and their associated benefits would probably have been within the "old" contract boundaries.

This reasoning does not yet take into account the fact that premiums tend to arrive earlier than benefits. This difference in timing leads to a bigger impact of adjusting the risk-free rate on the benefits than on the liabilities, potentially once again reversing the conclusion just drawn.

Therefore we shall illustrate things using a simple example.

Suppose we have the following insurance product:[3]

The insurer receives periodical premiums $P = 1000$, for eight years. These premiums are paid at the beginning of each year. The interest rate guarantee is r_g. Profit sharing (PS) is given, using a margin of μ. The PS percentage π_t in a certain year is calculated at the end of the year using the following equation:

$$\pi_t = (r_{\text{reference},t} - r_g - \mu)_+ \tag{C.1}$$

where $r_{\text{reference},t}$ is the reference rate in year t. The reference rate is a measure for the return achieved on the assets in front of the provisions in year t. The PS amount, which is calculated as $\Pi_t = {}_tV\pi_t$,[4] is used to constitute a PS provision, which we shall add to the "regular" provision to simplify things. Thus:

$${}_{t+1}V = ({}_tV)(1 + r_g) + P + \Pi_t \tag{C.2}$$

At the end of the 8 years, the insurer pays out ${}_8V$.

Let's take $\mu = 1\%$, and $r_g = 1.75\%$, and $r = 2\%$. We take $r_{\text{reference},t} = r_{\text{discount}}$, to exclude arbitrage opportunities. If we then perform discounting using a discount rate $r_{\text{discount}} = r_{\text{risk-free}} + s$, figure C.1 shows the evolution of calculated value as a function of s.

Note that the value can be expressed either as a negative present value of liabilities, or as a present value of margin amounts taken. Because the margin amounts the insurer can take are the maximum amounts (i.e. μ_tV) as soon as $s \geq 0.75$, these cash flows do not change anymore. Because they are discounted more severely when s rises, the value drops.

We can thus conclude that although in general, using higher rates to discount liabilities leads to a higher calculated value for the insurance company, this is not always the case, which was shown using a very basic example.

[3]Granted, there is no insurance component, so strictly speaking, this is not an insurance product, but adding an insurance component would not change the conclusions.

[4]${}_tV$ is the technical provision in year t.

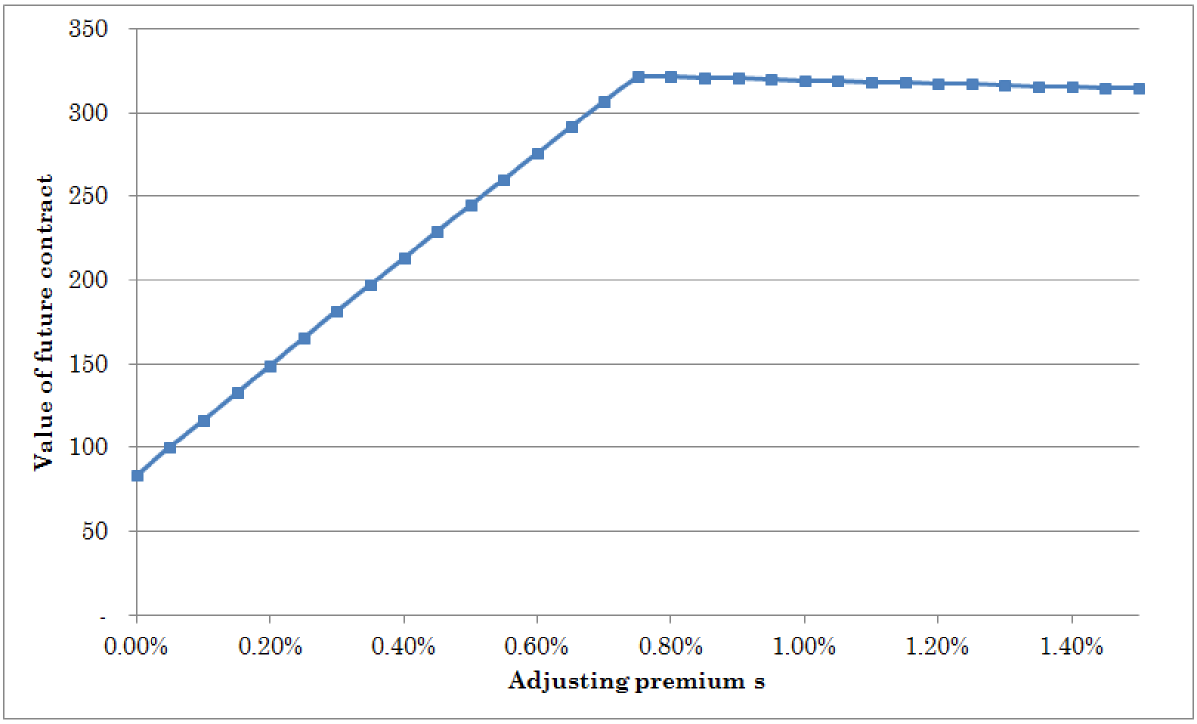

Figure C.1: Adding premiums to the risk-free rate does not necessarily increase value

Appendix D

Additional figures

This appendix provides some additional figures, which were felt unneccessary to include in the main text.

Figure D.1 shows empirical percentiles of a risk-neutral distribution, obtained using 1000 risk-neutral simulations. The risk-neutral distribution is based upon a Hull-White model (see equation 2.7), using $\alpha = 5.08\%$ and $\sigma = 1.21\%$.

Figure D.2 depicts the evolution of the spread between the 3 month USD LIBOR and the 3 month Overnight Indexed Swap (OIS) rate since the beginning of 2005. Until July 2007, the spread is rather constant. From August 2007 until the end of 2009, the spread has been much higher. The LIBOR OIS spread end of April 2012 is 33bp.

Figure D.3 depicts the evolution of the liquidity premium when calculated as described by the Solvency II QIS5 Technical Specifications [60]. The swap spread, which is the difference between the swap rates and high quality European government bonds[1] is deducted from the spread between corporates and the same government bonds, to obtain the spread between the assumed risk-free swap curve and the corporate bonds. Of this spread, 40bp is estimated to be due to default probability, and of the remainder, 50% is estimated to be due to illiquidity, see also chapter 4.

[1] E.g. German Bunds.

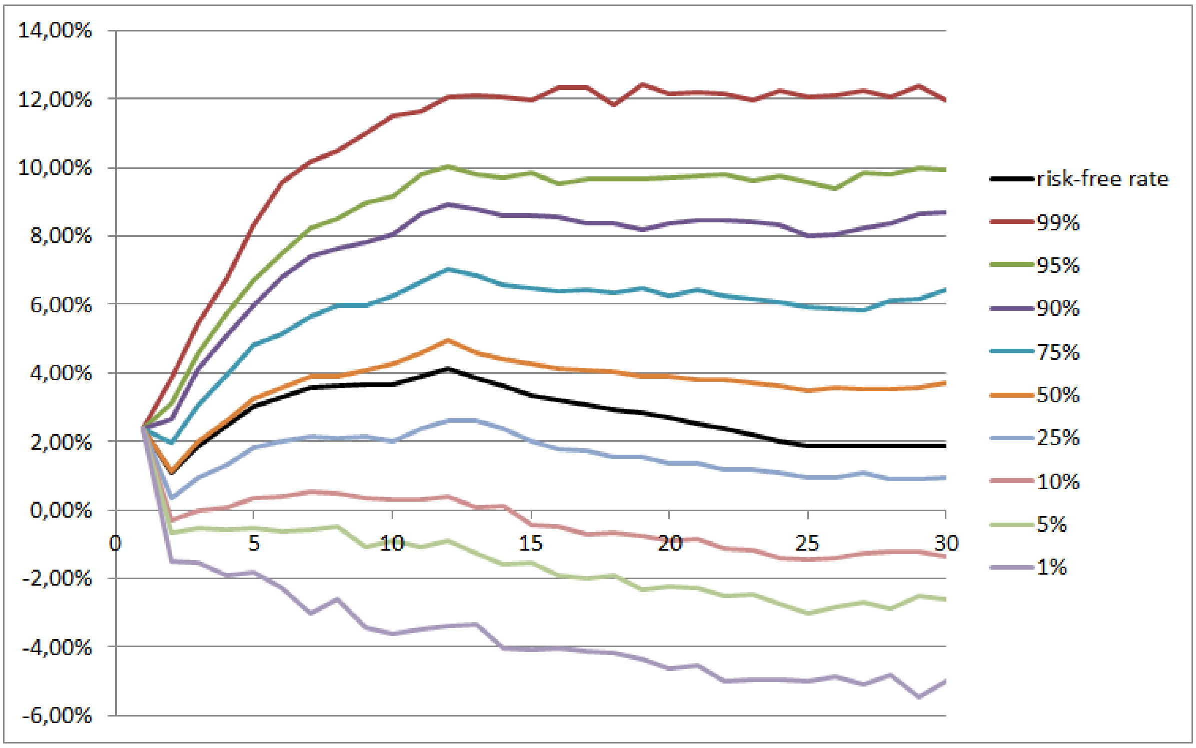

Figure D.1: A risk-free term structure (forward one year rates) at 2011 year-end to discount life insurance liabilities, and the percentiles of a risk-neutral scenario distribution obtained using 1000 samples of a Hull-White model based upon this term structure. $\alpha = 5.08\%$, $\sigma = 1.21\%$

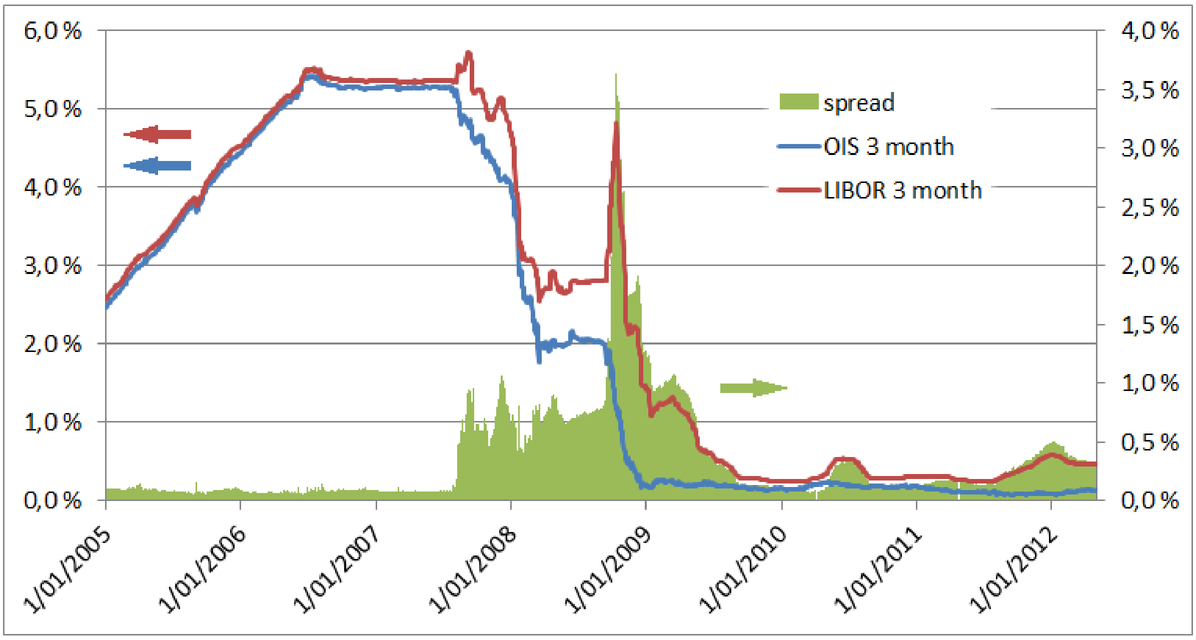

Figure D.2: Evolution of the USD LIBOR-OIS (3 month) spread. Spread is plotted w.r.t. secondary (right) axis. Source: Bloomberg.

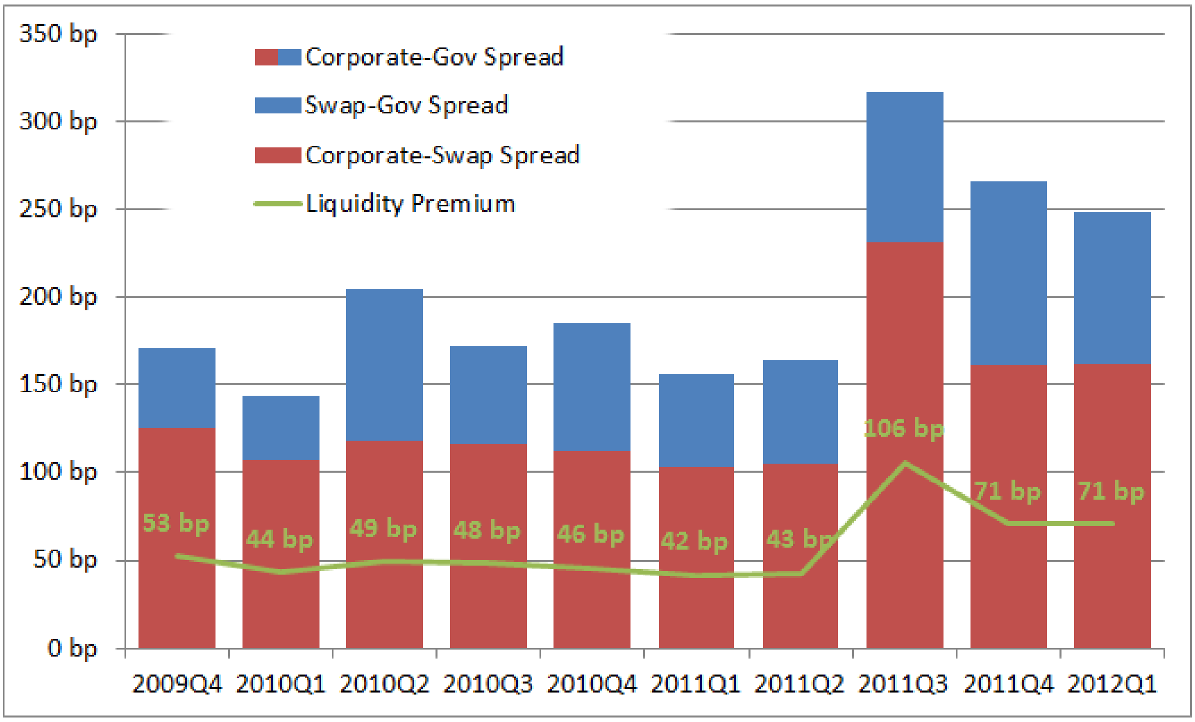

Figure D.3: Evolution of the liquidity premium since the end of 2009, the valuation date of QIS5. Sources: Markit and Bloomberg.

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